# EECE 301
# Signals & Systems

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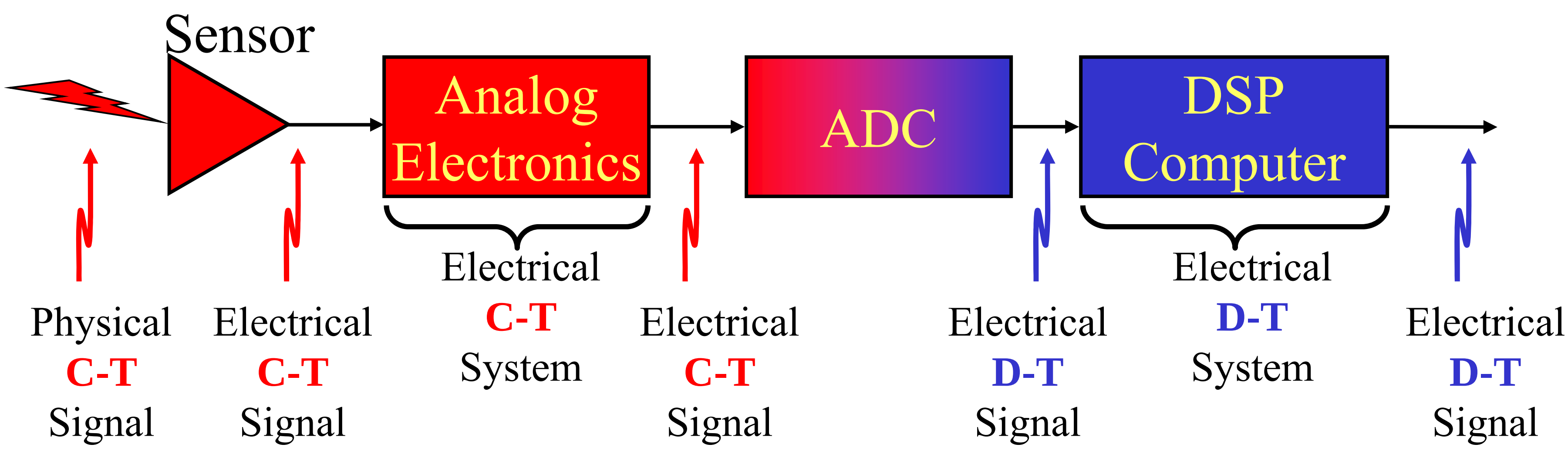
## **Note Set #3**

- What are Discrete-Time Signals???

# 1.2 Discrete-Time (D-T) Signals

Recall from earlier that a common scenario in today's electronic systems is to do most of the processing of a signal using a computer.

A computer can't directly process a C-T signal but instead needs a stream of numbers… which is a **D-T signal**.



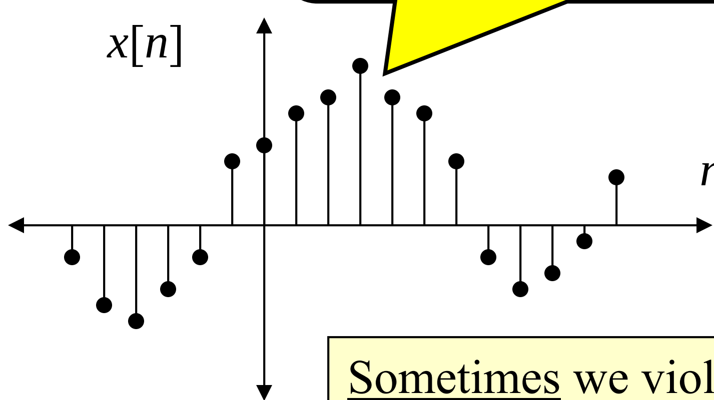
# What is a discrete-time (D-T) signal?

A discrete time signal is a sequence of numbers indexed by integers

Example: $x[n] \longrightarrow n = \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$

Brackets indicate discrete-time signal. Recall… we used parentheses to indicate a C-T signal.

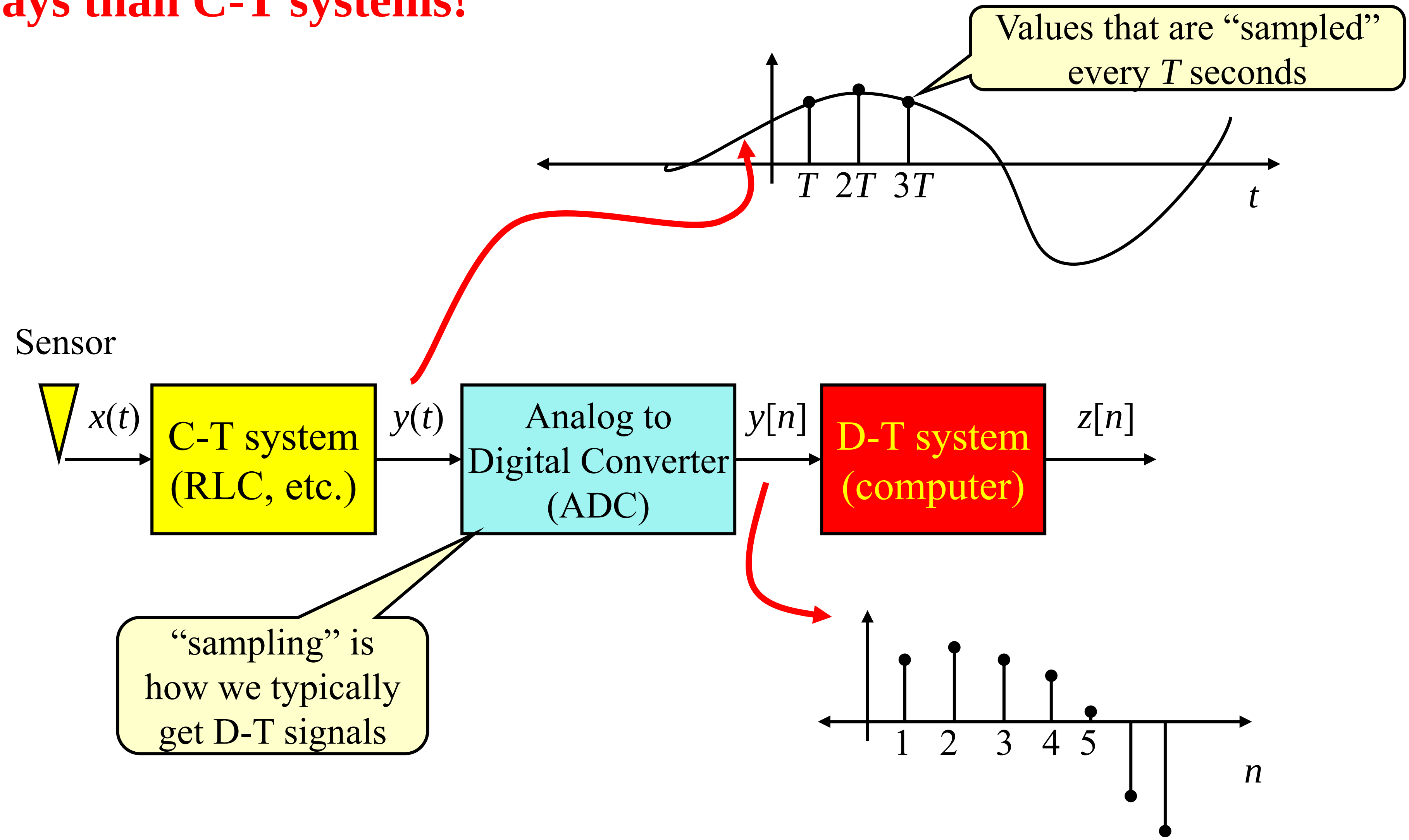
A stem plot emphasizes that the signal does not exist in-between integer n values




Sometimes we violate this and plot with line segments connecting the dots.

Remember: for our point of view, D-T signals are needed to allow us to process signals (i.e. information) using D-T systems rather than only Continuous-Time systems

**D-T systems allow us to process information in <u>much</u> more amazing ways than C-T systems!**



# Mathematical Expression for Sampling:

**The D-T signal $y[n]$ is related to the C-T signal $y(t)$ by:**

$$y[n] = y(t)\big|_{t=nT} = y(nT)$$

$T$ = time spacing between samples (seconds)

$\boxed{T \text{ is “sampling interval”}}$

$1/T$ = sampling rate ($F_s$) in samples/second

$\boxed{F_s \text{ is “sampling rate”}}$

**<u>Ex</u>**: CD audio is sampled at 44,100 samples per second

$\Rightarrow T = 1/44{,}100 \cong 22.68\ \mu\text{sec}$

**<u>Major Question</u>**: How fast should we sample a specific signal?

(We can't answer that until we cover Fourier Transforms!!)

**<u>Hint</u>**: You may know that humans can't hear frequencies above approximately 20kHz. Therefore, audio signals typically are limited to have no frequencies above 20kHz. Note that CD audio uses a sampling rate that is slightly more than twice this "highest" frequency!!

# Digital Signals

-A practical ADC not only gives a D-T signal but also one that is "digital"

Can take values only from a finite set

Don't confuse "digital" with logic signals!!!

-An ADC represents each sample $y[n]$ using a finite number of bits (typically 8 to 16 bits/sample)

-ADC's have a min/max input voltage

-If the ADC uses $B$ bits per sample and V = max = - min volts

Then there are $2^B$ levels or values that are spaced $\dfrac{2V}{2^B} = V2^{(-B+1)}$ volts apart

This "quantization" of values causes degradation in the quality of the representation of $y[n]$

**<u>For this course</u> we will ignore the quantization!**
**Our D-T signals can take on any value!**

# Some Common D-T Signals

Much of what we learned about C-T signals carries over to D-T signals

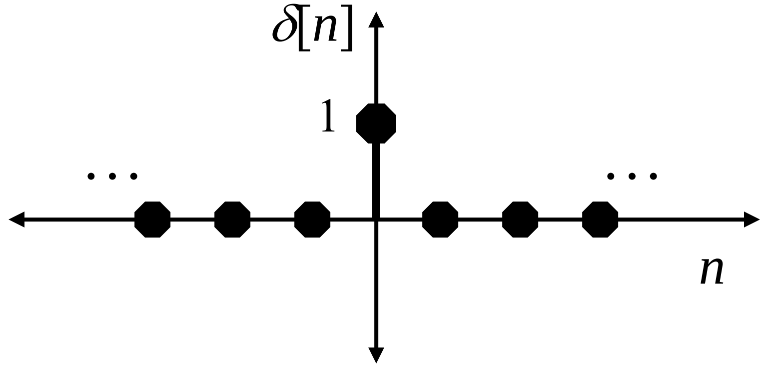
The **D-T Unit Step** is defined in an obviously similar way that the C-T Unit Step was defined. The D-T unit step is just a sampled version of the C-T unit step

The same holds true for the **D-T Unit Ramp.**

**However… there are a Few Exceptions…**

**Unit Pulse**: $\delta[n]$

"**D-T Impulse**" or "**D-T Delta**"




$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

Note: $\delta[n]$ is not a sampled version of $\delta(t)$

# Sifting Property for D-T Delta Function

Note: $\delta[n]$ works inside summations the same way

$\delta(t)$ works inside integrals

$$\sum_{n=-\infty}^{\infty} \delta[n] = 1$$

Compare

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

$$\sum_{n=-\infty}^{\infty} x[n]\delta[n-n_0] = x[n_0]$$

Compare

$$\int_{-\infty}^{\infty} x(t)\delta(t-t_0)\,dt = x(t_0)$$

# D-T Rectangular Pulse

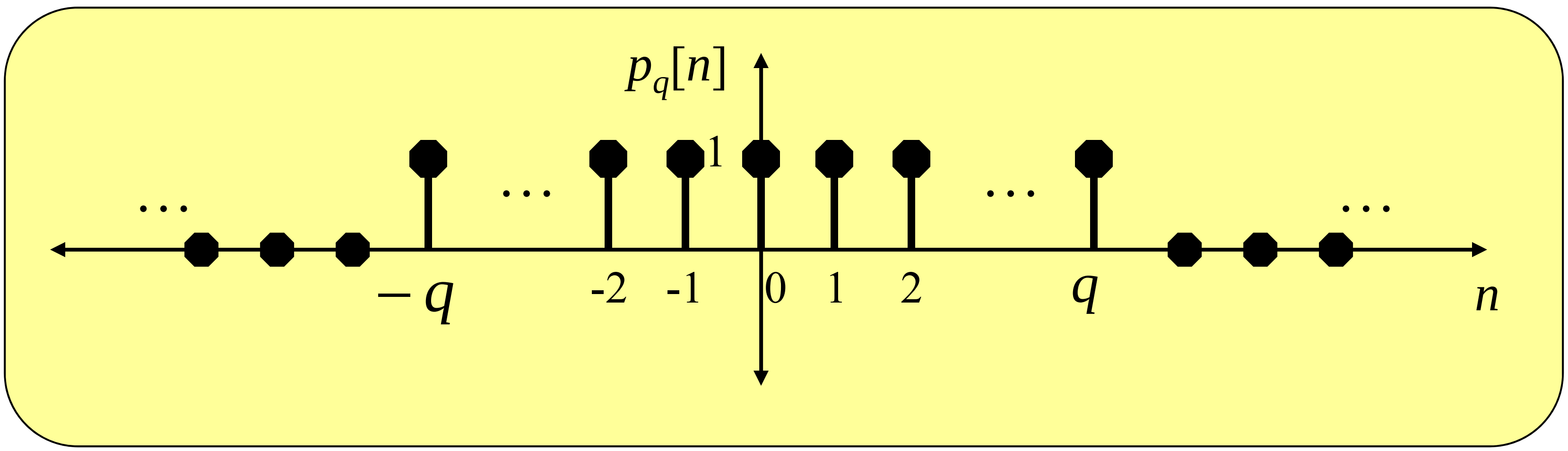
Often in practice we need to use pulses to model real-world signals…

One definition of DT version of this is as follows.

Let $q$ be an integer, then a centered pulse of $2q+1$ samples is

$$p_q[n] = \begin{cases} 1, & n = -q, \ldots, -1, 0, 1, \ldots, q \\ 0, & otherwise \end{cases}$$

$q$ specifies pulse +/- extent



# D-T Sinusoids

$$x[n] = A\cos(\Omega n + \theta)$$

Use "upper case omega" for frequency of D-T sinusoids

**What is the unit for Ω?**

Anything inside a sine, cosine, or tangent must be in radians… NOT in degrees!!

$\Omega n + \theta$ must be in radians $\Rightarrow$ $\Omega n$ in radians

$\Rightarrow$ $\Omega$ is "how many radians jump for each sample"

$\Rightarrow$ $\boxed{\Omega \text{ is in radians/sample}}$

Other than the above difference… D-T sinusoids are pretty much like C-T sinusoids.