EECE 301
Signals & Systems
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Note Set #12

- C-T Signals: FT Table Results

Table of Common Fourier Transform Results

We have just found the FT for a common signal…

$$p_\tau(t) = \begin{cases} 1, & -\dfrac{\tau}{2} \le t \le \dfrac{\tau}{2} \\ \\ 0, & otherwise \end{cases} \qquad \longleftrightarrow \qquad P_\tau(\omega) = \tau \, \text{sinc}\left(\dfrac{\omega\tau}{2\pi}\right)$$

We derived that result by directly applying the integral form of the FT to the given signal equation.

For the common "textbook" signals this has already been done… and the results are available in tables published in books and on-line

You should study the table provided…
*   If you encounter a time signal or FT that is on this table you should recognize that it is on the table without being told that it is there.
*   You should be able to recognize entries in graphical form as well as in equation form.
*   Later we'll learn about some "FT properties" that will expand your ability to apply these entries on the FT Table

In the real-world, engineers use these table results to understand basic ideas and concepts and to think through how things work in principle!

So… next we'll look at some of the more important entries in the table provided…

Decaying Exponential

As we'll see later… this signal naturally occurs in lots of real-world places!

$$x(t) = e^{-bt}u(t)$$
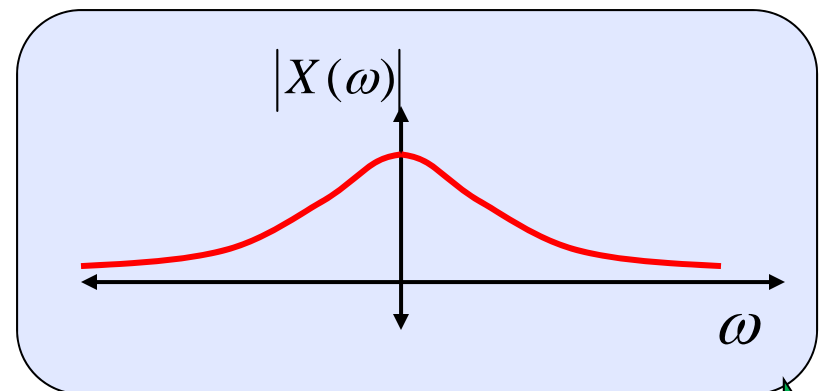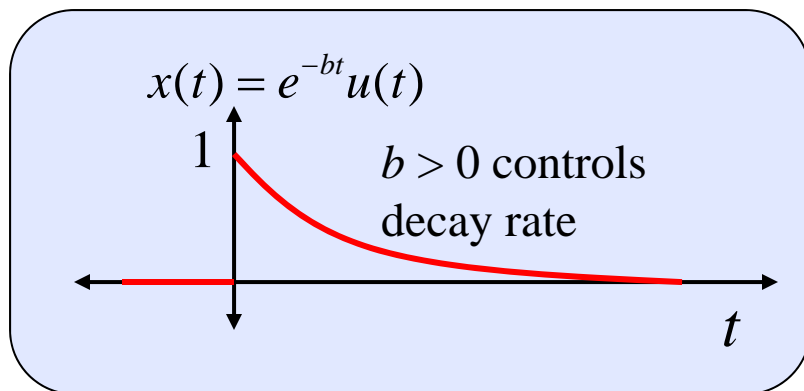
For $b > 0$

$$X(\omega) = \frac{1}{b + j\omega}$$

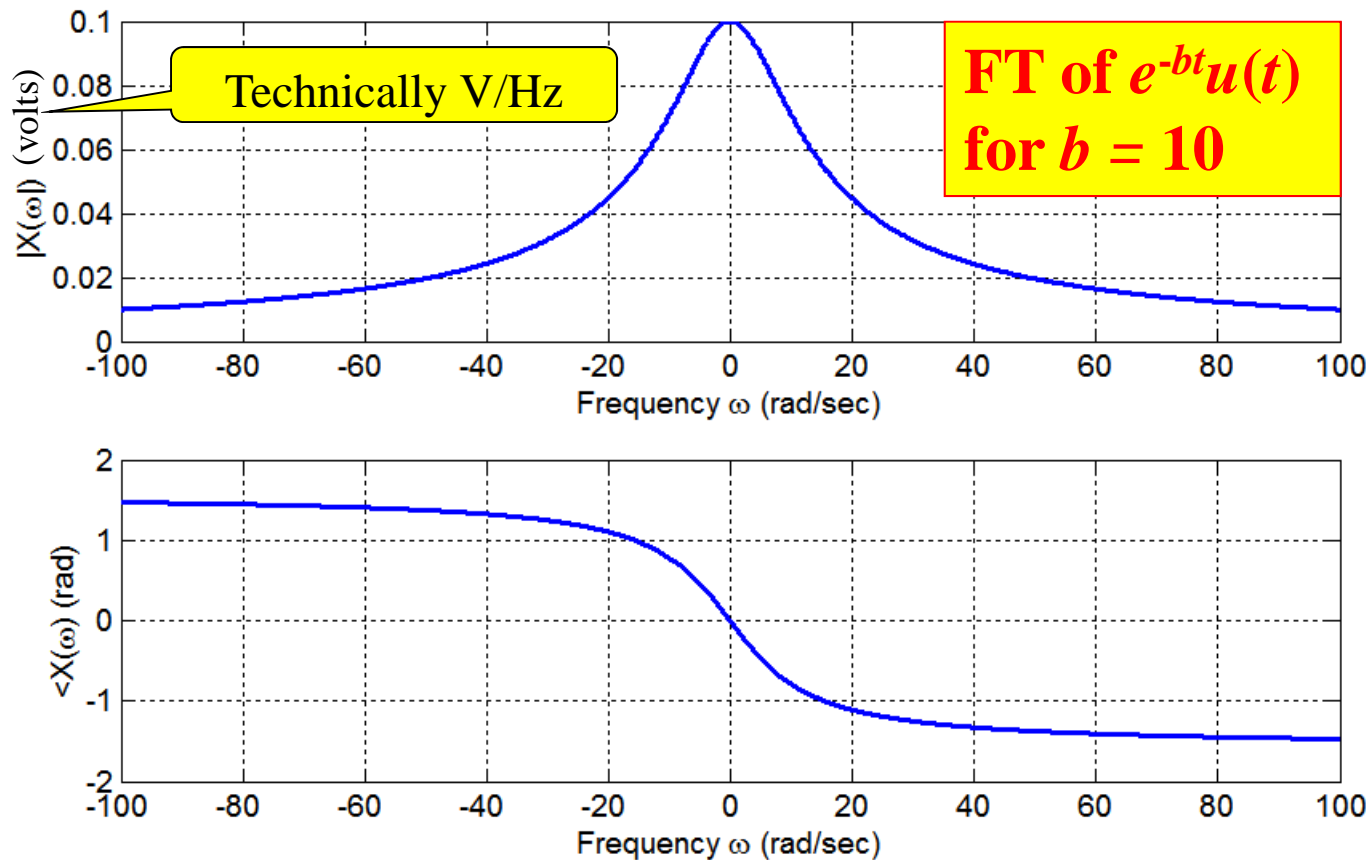(Complex Valued)

$$|X(\omega)| = \frac{1}{\sqrt{b^2 + \omega^2}}$$ Magnitude

$$\angle X(\omega) = -\tan^{-1}\left(\frac{\omega}{b}\right)$$ Phase

$x(t) = e^{-bt}u(t)$

1

$b > 0$ controls decay rate

t

$|X(\omega)|$

ω

Can Use Matlab to Make Plots of FT Results

Technically V/Hz

FT of $e^{-bt}u(t)$ for $b = 10$

Note that magnitude plot has <u>even</u> symmetry

Note that phase plot has <u>odd</u> symmetry

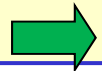**True for <u>every</u> real-valued signal**

MATLAB Commands to Compute FT

```
w=-100:0.2:100;
b=10;
X=1./(b+j*w);
```
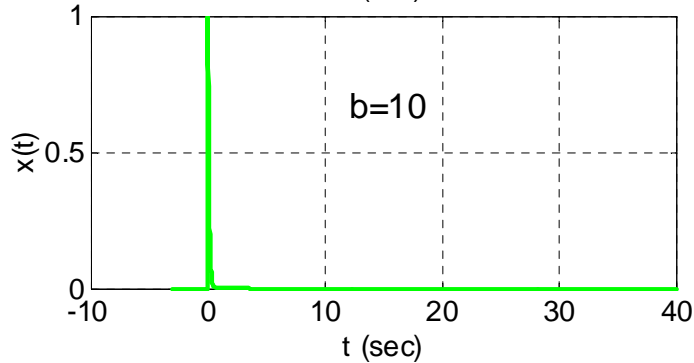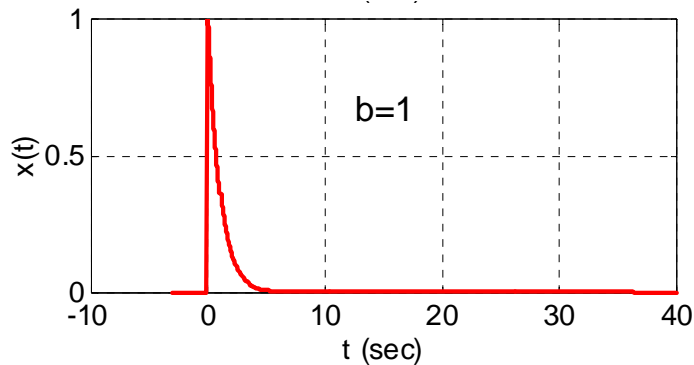
Plotting Commands

```
subplot(2,1,1); plot(w,abs(X))
xlabel('Frequency \omega (rad/sec)')
ylabel('|X(\omega|) (volts)'); grid
subplot(2,1,2); plot(w,angle(X))
xlabel('Frequency \omega (rad/sec)')
ylabel('<X(\omega) (rad)'); grid
```
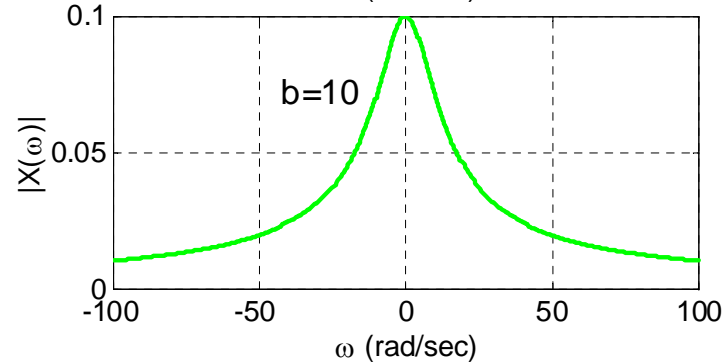
Effect of Exp. Decay Rate b on FT Magnitude
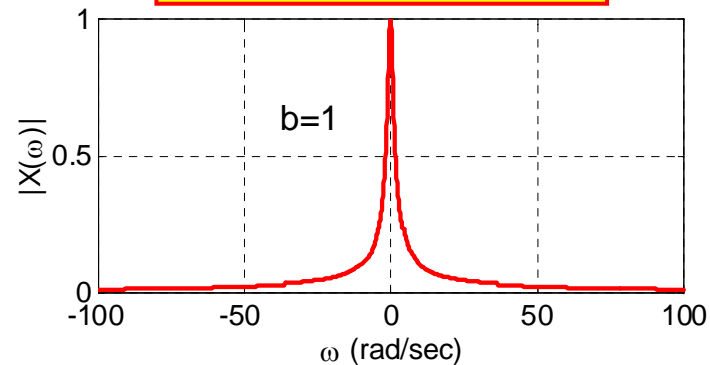
Time Signal

$$x(t) = e^{-bt}u(t)$$

FT Magnitude

$$\left|X(\omega)\right| = \frac{1}{\sqrt{b^2 + \omega^2}}$$

Note: As b increases…
1. Decay rate in time signal increases
2. High frequencies in Fourier transform are more prominent.

Short Signals have FTs that spread more into High Frequencies!!!

Some Important Signals & Their FTs (see Table for More!)

$$1, \quad -\infty < t < \infty \iff 2\pi\delta(\omega)$$

$$u(t) \iff \pi\delta(\omega) + 1/j\omega$$

$$-0.5 + u(t) \iff 1/j\omega$$

$$\delta(t) \iff 1, \quad -\infty < \omega < \infty$$

$$\cos(\omega_o t) \iff \pi[\delta(\omega + \omega_o) + \delta(\omega - \omega_o)]$$

$$\sin(\omega_o t) \iff j\pi[\delta(\omega + \omega_o) - \delta(\omega - \omega_o)]$$

$$e^{j\omega_o t} \iff 2\pi\delta(\omega - \omega_o), \quad \omega_o \text{ real}$$

FT of Periodic Signal

Note that we have now used the FT to analyze cosine and sine… which are _PERIODIC_ signals!!! Before we used the Fourier _Series_ to analyze _periodic_ signals… Now we see that we can also use the Fourier _Transform!_

If $x(t)$ is periodic then we can write the FS of it as: $x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$
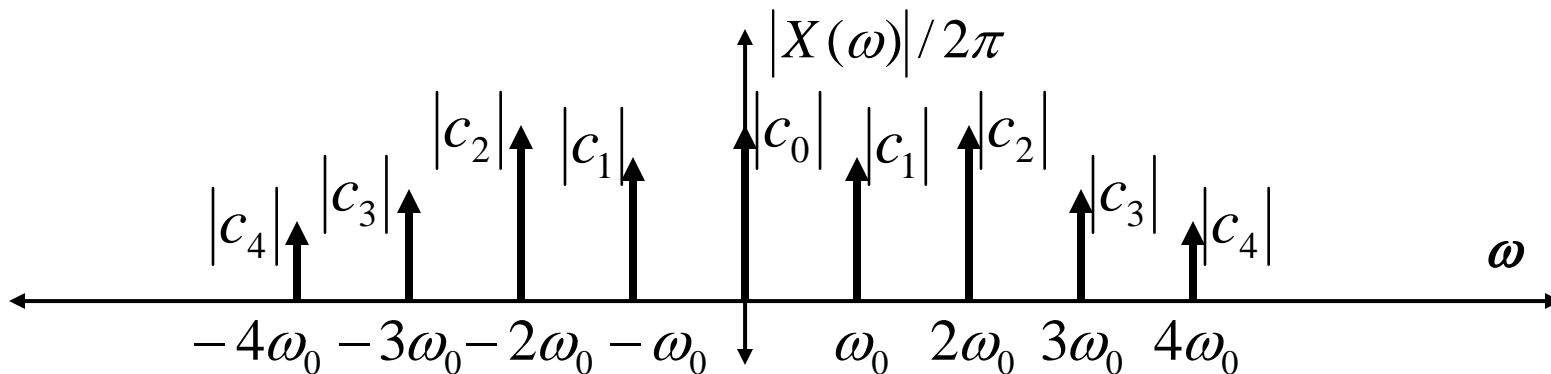
Now we can take the FT of both sides of this: $\mathscr{F}\{x(t)\} = \mathscr{F}\left\{ \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} \right\}$

$$= \sum_{k=-\infty}^{\infty} c_k \underbrace{\mathscr{F}\{e^{jk\omega_0 t}\}}_{2\pi\delta(\omega - k\omega_0)}$$

FT of a Periodic Signal

$$X(\omega) = \sum_{k=-\infty}^{\infty} 2\pi c_k \delta(\omega - k\omega_0)$$

Note: the FT is a bunch of delta functions with "weights" given by the FS coefficients!

$$|X(\omega)|/2\pi$$

$|c_4|$ $|c_3|$ $|c_2|$ $|c_1|$ $|c_0|$ $|c_1|$ $|c_2|$ $|c_3|$ $|c_4|$ ω

$-4\omega_0$ $-3\omega_0$ $-2\omega_0$ $-\omega_0$ ω_0 $2\omega_0$ $3\omega_0$ $4\omega_0$

So the FT of a periodic signal is zero except at multiples of the fundamental frequency ω_0, where you get impulses.

We call these spikes "Spectral Lines"

Note that if we start with the Amplitude-Phase Trig form we end up with the same result for the FT

$$x(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos\left(k\omega_0 t + \theta_k\right)$$

For each cosine term we get two deltas (a positive frequency & negative frequency):

$$\cos(\omega_o t + \theta) \iff \pi\left[e^{-j\theta}\delta(\omega+\omega_o) + e^{j\theta}\delta(\omega-\omega_o)\right]$$