BINGHAMTON UNIVERSITY
State University of New York

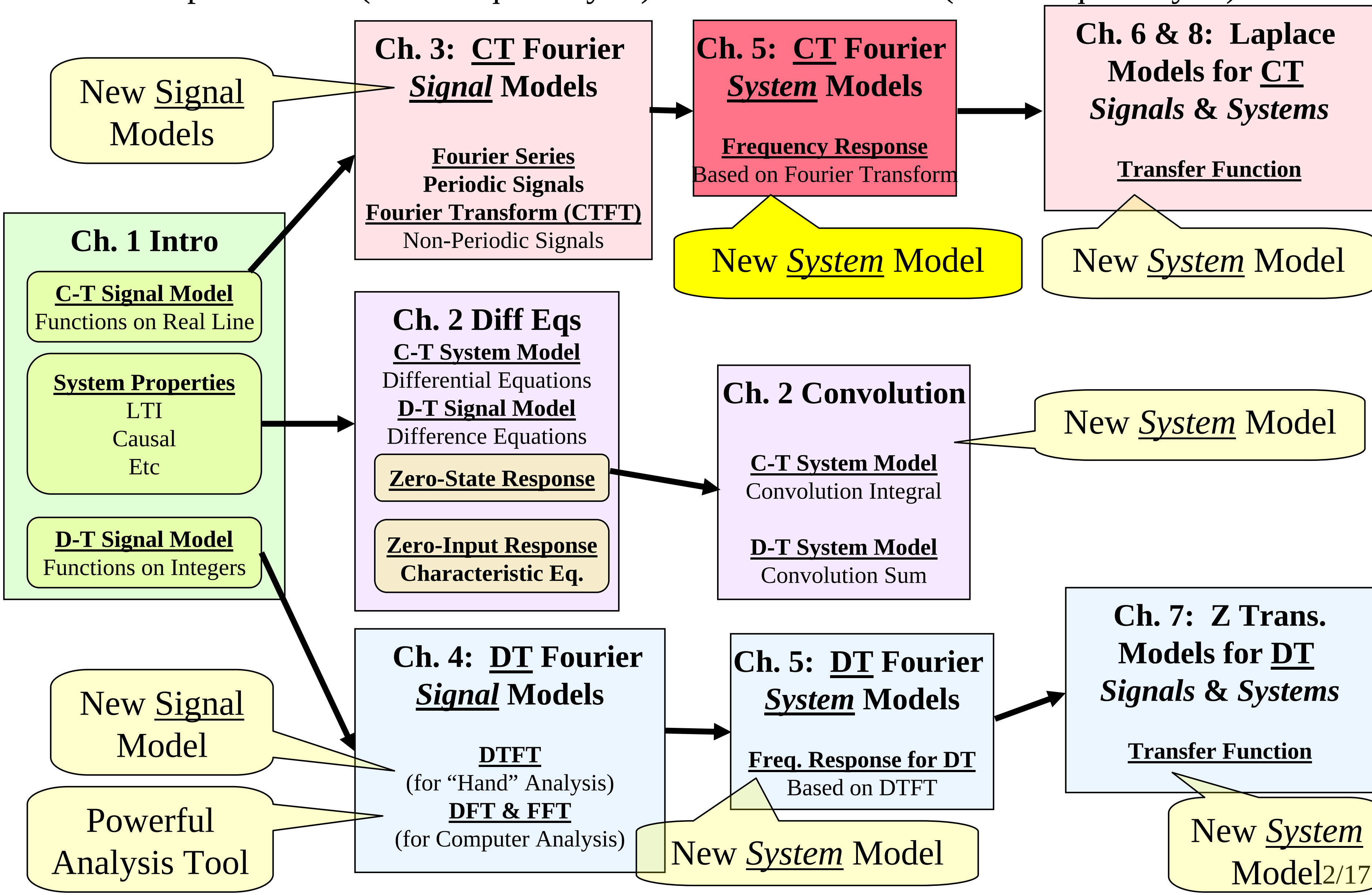
EECE 301
Signals & Systems
Prof. Mark Fowler

Note Set #19

- C-T Systems: Frequency-Domain Analysis of Systems
- Reading Assignment: Section 5.2 of Kamen and Heck

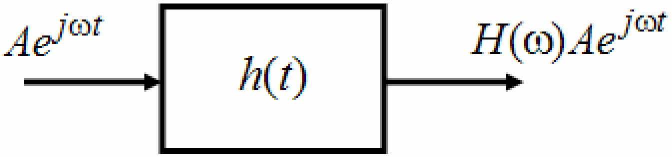
Course Flow Diagram

The arrows here show conceptual flow between ideas. Note the parallel structure between the pink blocks (C-T Freq. Analysis) and the blue blocks (D-T Freq. Analysis).



5.2 Response to Aperiodic Signals

Recall:



where

$$H(\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}\,dt$$

Thus: Frequency Response = FT{Impulse Response}

-<u>Impulse Response</u> $h(t)$ is a <u>time-domain description</u> of the system

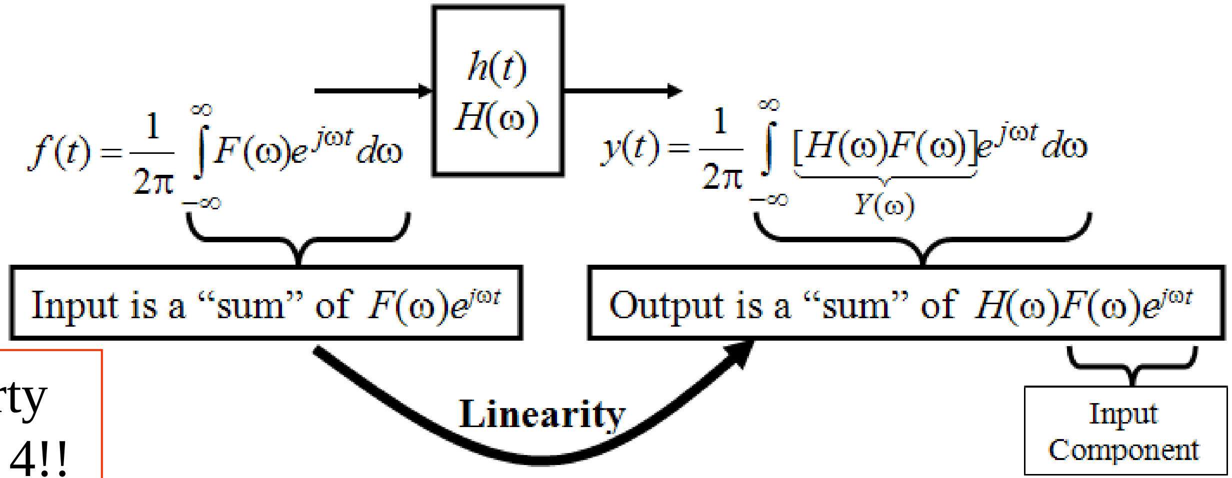
-<u>Frequency Response</u> $H(\omega)$ is a <u>frequency-domain description</u> of the system

<u>Recall that:</u>

Because $h(t)$ and $H(\omega)$ form a FT pair, one completely defines the other.

$h(t)$ and convolution completely describe the zero-state response of an LTI to an input – i.e. $h(t)$ completely describes the system.

<u>Thus: $H(\omega)$ must also completely describes the LTI system</u> **HOW????**



Conv. Property from chapter 4!!

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**** This says: FT of Output = [FT of Input]×[Freq Resp] ****

****

<u>"Proof"</u>

Step 1: Think of the input as a sum of complex sinusoids

-Each component $= F(\omega)e^{j\omega t}$

Step 2: We know how each component passes through an LTI

-This is the idea of <u>frequency response</u>

- $H(\omega)F(\omega)e^{j\omega t}$ is the <u>out. component</u> that is due to the <u>input component</u> $F(\omega)e^{j\omega t}$

Step 3: Exploit System Linearity (again – Step 2 was the first time)

-Total output is a sum of output components $y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} [H(\omega)F(\omega)]e^{j\omega t}\,d\omega$

Input-Output Relationship Characterized Two Ways

1. Time-Domain: $y(t) = h(t)*f(t)$

2. Freq-Domain: $Y(\omega) = H(\omega)F(\omega)$

Given input $f(t)$ and impulse response $h(t)$, to analyze the system we could either:

1. Compute the convolution $h(t)*f(t)$

or…

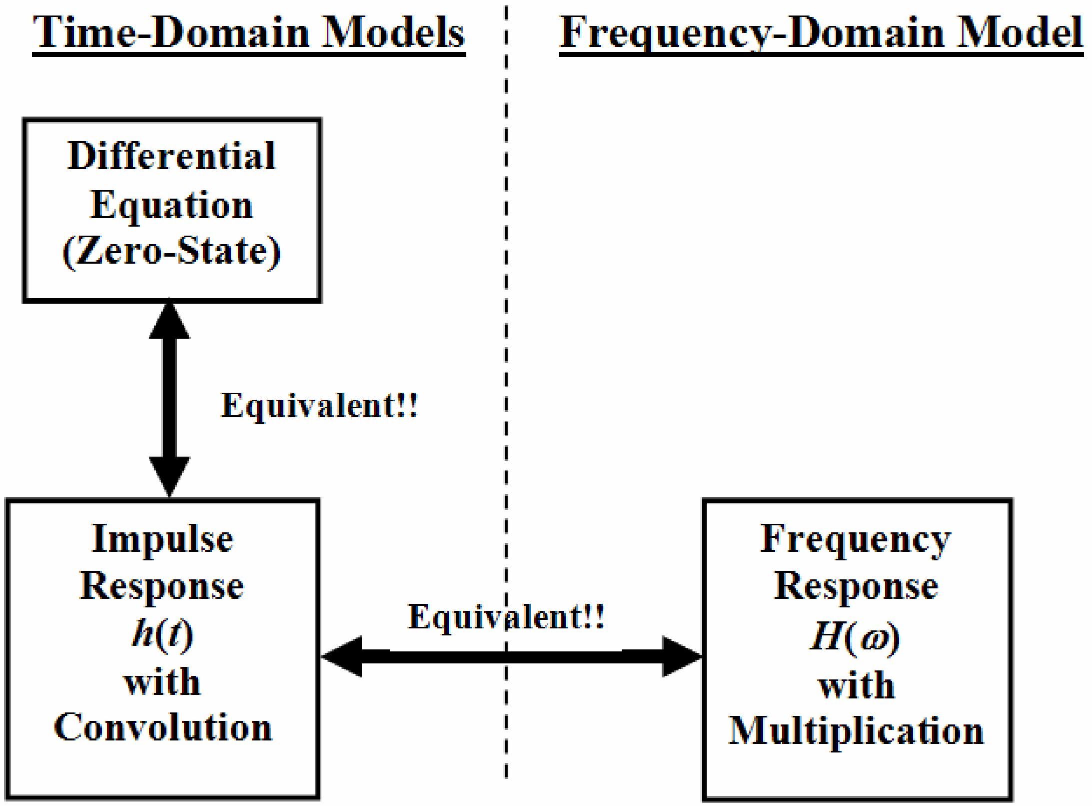
2. Do the following:
   (a) Compute $H(\omega)$ & compute $F(\omega)$
   (b) Compute the product $Y(\omega) = H(\omega)F(\omega)$
   (c) Compute the IFT: $y(t) = \mathscr{F}^{-1}\{ H(\omega)F(\omega)\}$

Method #2 (Freq-Domain Method) may not be necessarily easier, but it usually provides a lot more insight than Method #1!!!!

From the Freq-Domain view we can see how $H(\omega)$ boosts or cuts the amounts of the various frequency components

Relationships between various modeling methods

Recall: we are trying to find ways to model… CT Linear Time-Invariant Systems in Zero-State



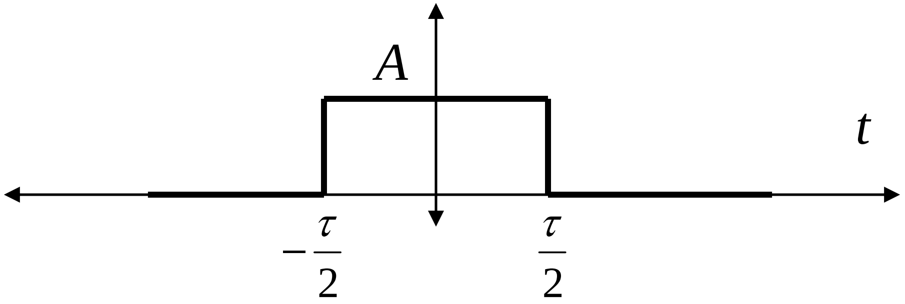
Since these are all equivalent…we can use any or all of them to solve a given problem!!

Example

Scenario: You need to send a pulse signal into a computer's interface circuit to initiate an event (e.g. "next PTT slide")

Q: What kind of signal should you use?

Possibility: A rectangular pulse: $Ap_\tau(t)$

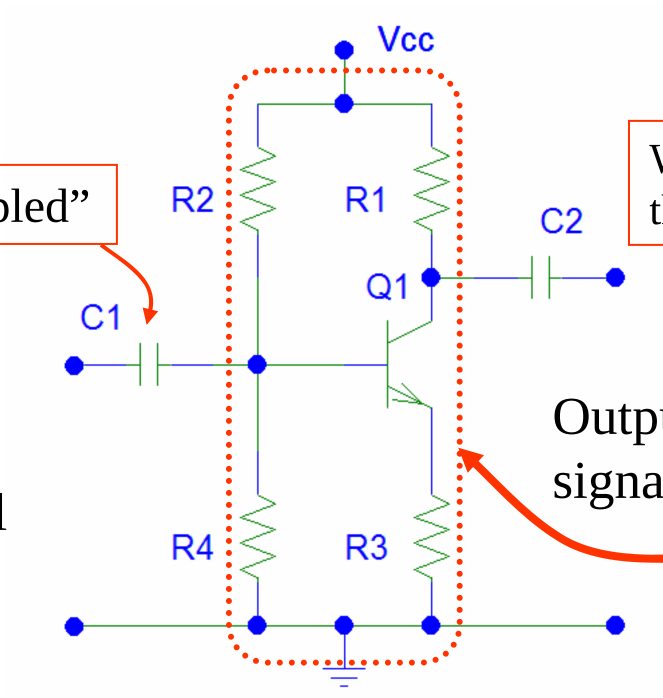


Q: Will this work?

It depends on the interface circuitry already in the computer!

Suppose the interface circuitry consists of an "AC Coupled" transistor amplifier as shown below

“AC coupled”



We’ll ignore the effects of this capacitor in our analysis

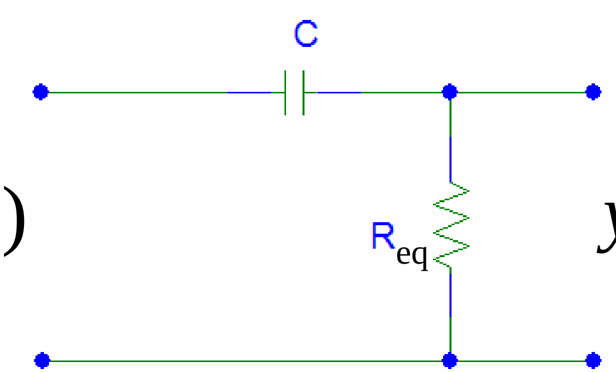
Input signal

Output signal

Model this as an equivalent Input Impedance… simplify here: R_{eq}

<u>“Equivalent Circuit Model”</u>

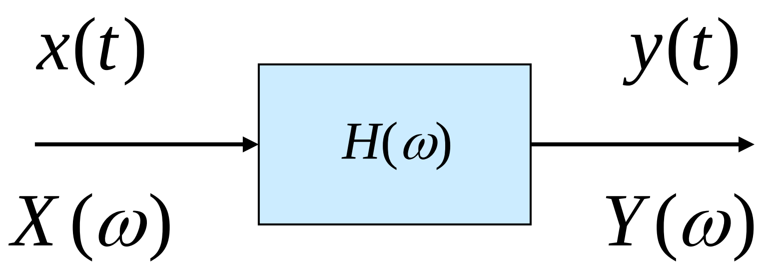
$x(t)$



$y(t)$

Now we need to find the System Model viewpoint!

"Equivalent System Model"



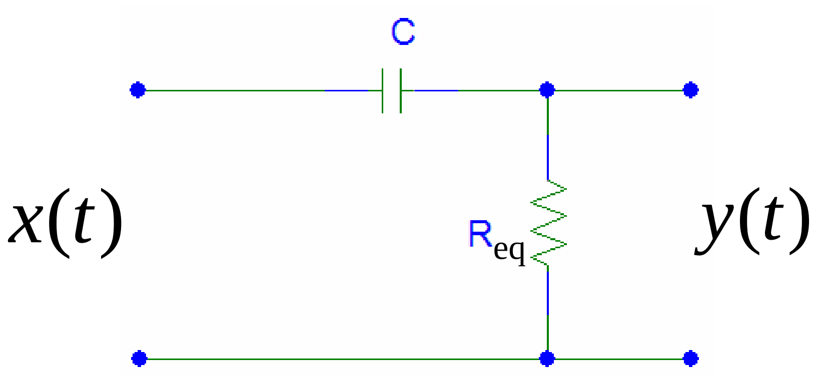
Actually… one LIKE it!

What is $H(\omega)$??

Use Sinusoidal Analysis to find it… we did that once already for this circuit…

Use Phasors, Impedances, and Voltage Divider:

$$\vec{V}_0 = \left[\frac{R_{eq}}{R_{eq} + \frac{1}{j\omega C}} \right] \vec{V}_i$$

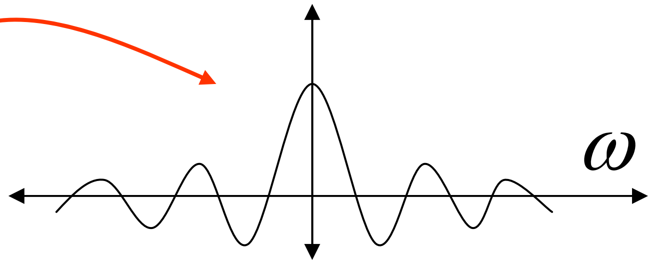


$$\Rightarrow \quad H(\omega) = \frac{j\omega R_{eq} C}{1 + j\omega R_{eq} C}$$

Now…what does the input pulse look like in the frequency domain?

From FT table:

$$p_\tau(t) \quad \leftrightarrow \quad \underbrace{\tau \,\text{sinc}\left(\frac{\tau\omega}{2\pi}\right)}_{X(\omega)}$$



So the output FT looks like: $Y(\omega) = H(\omega)X(\omega) = \underbrace{\tau \,\text{sinc}\left(\frac{\tau\omega}{2\pi}\right)\left[\frac{j\omega R_{eq}C}{1 + j\omega R_{eq}C}\right]}$

Now how do we find *y*(*t*)? $\quad y(t) = \mathcal{F}^{-1}\{Y(\omega)\}$

So find IFT of this…

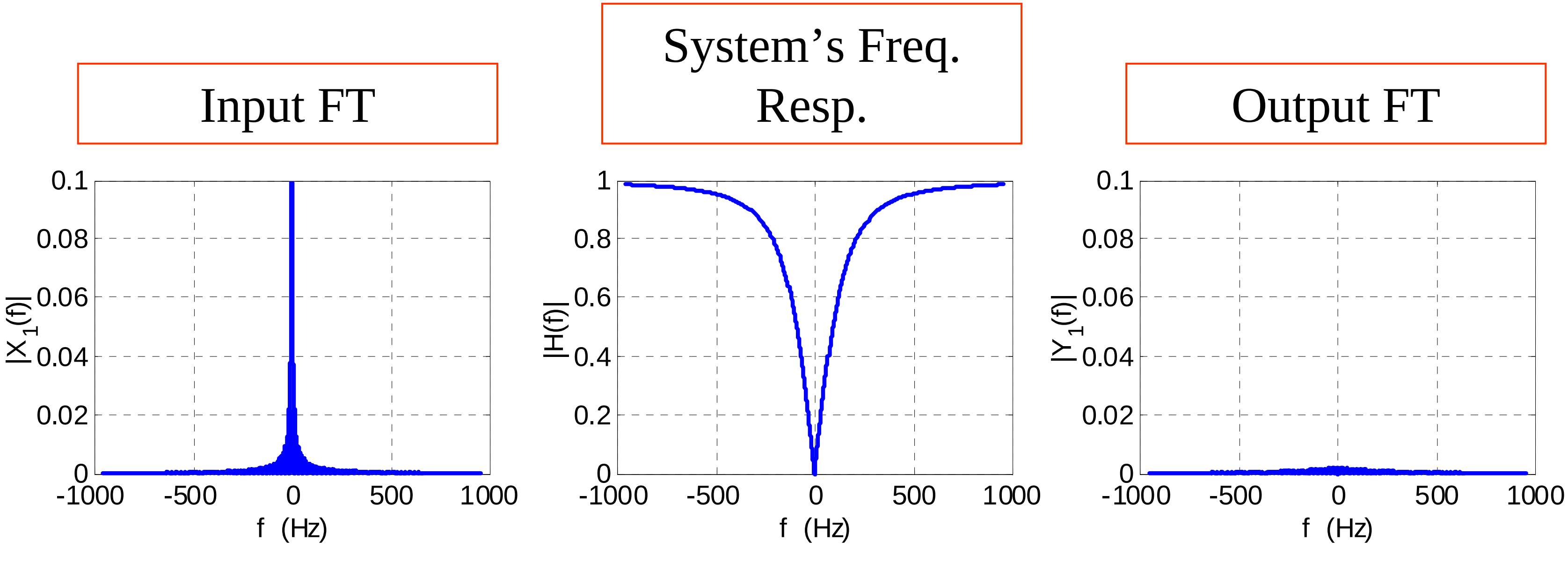
YUCK!!! HARD!!!

Well…do we need to “go back to the time domain”? NO!

Just look at $Y(\omega)$ and see what it tells

Think Parseval’s theorem

The plots below show that very little energy gets through the system



So this pulse signal is not usable here because very little of its energy gets through the interface circuitry!!!

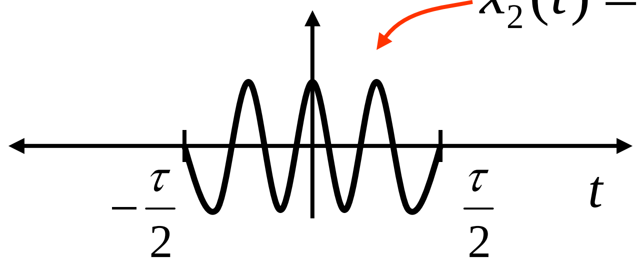
The problem lies in that $|H(\omega)|$ is small where $|X(\omega)|$ is big

(and vice versa)

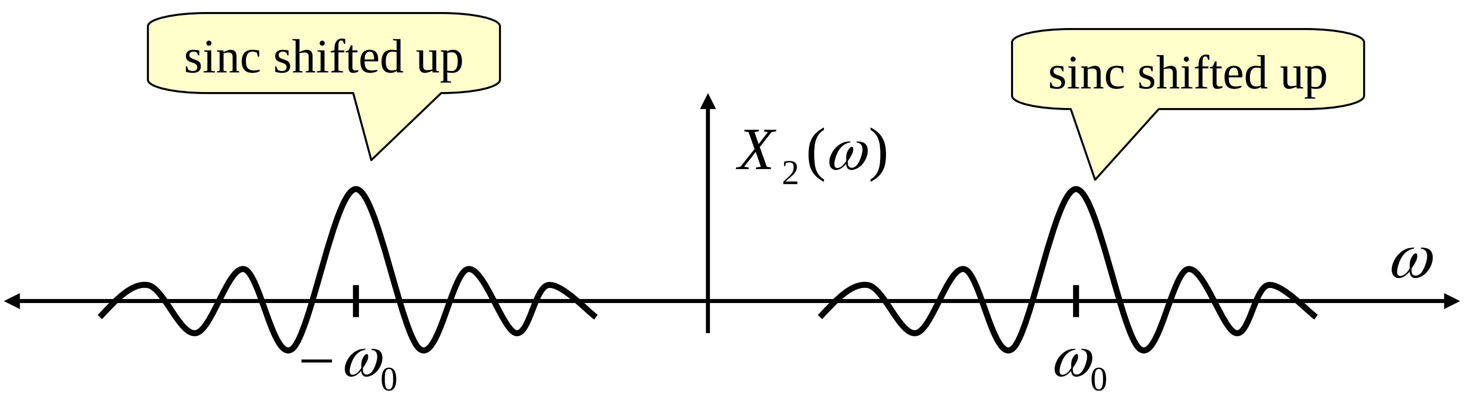
$\Rightarrow$ Pick an $X(\omega)$ that does not do that!!

Use a pulse that is "Modulated Up" to where $|H(\omega)|$ allows it to pass

$$x_2(t) = p_\tau(t)\cos(\omega_0 t)$$



"A modulated pulse"

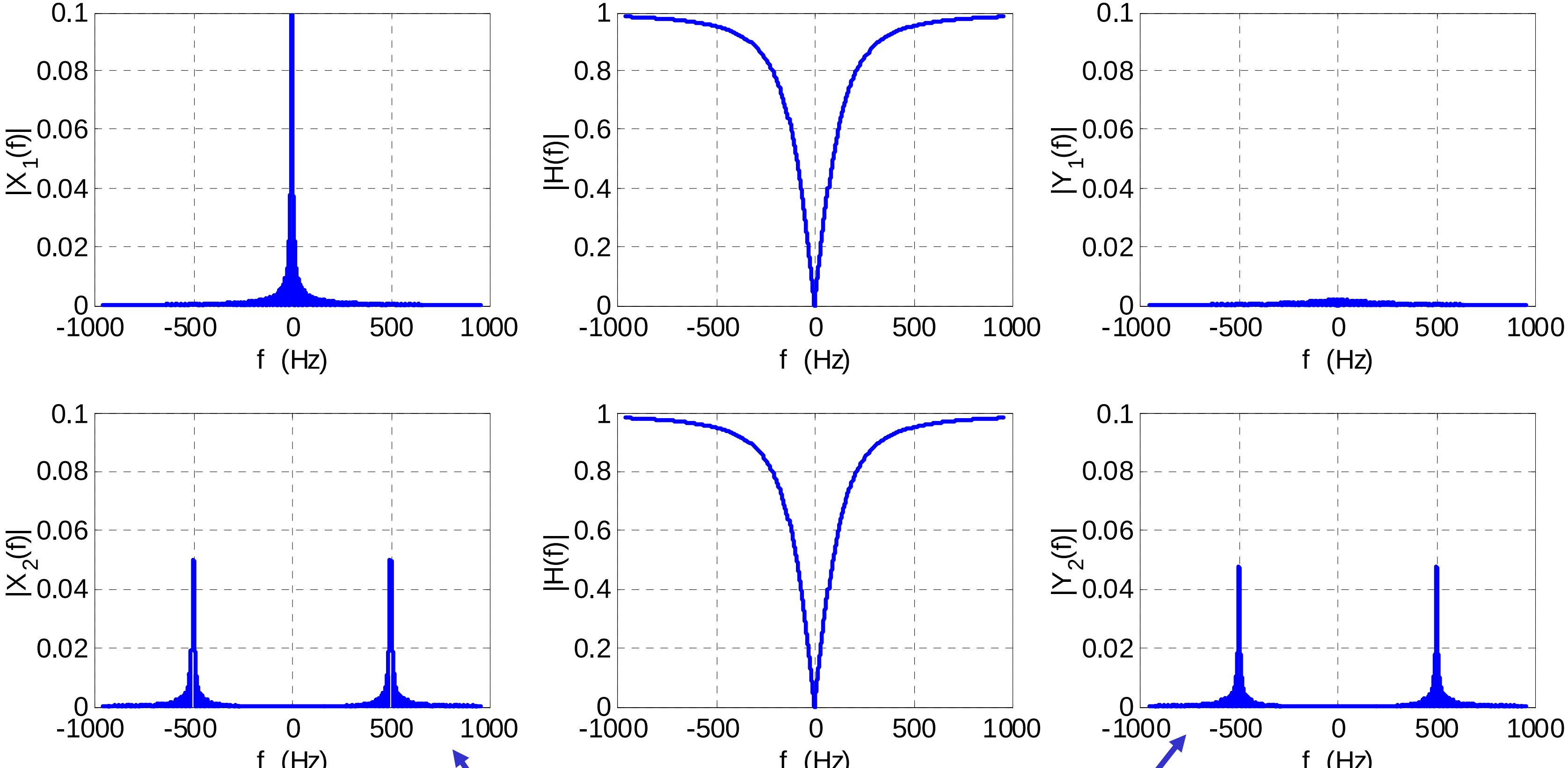


See actual plots on next page

Original Input FT

System’s Freq. Resp.

Original Output FT



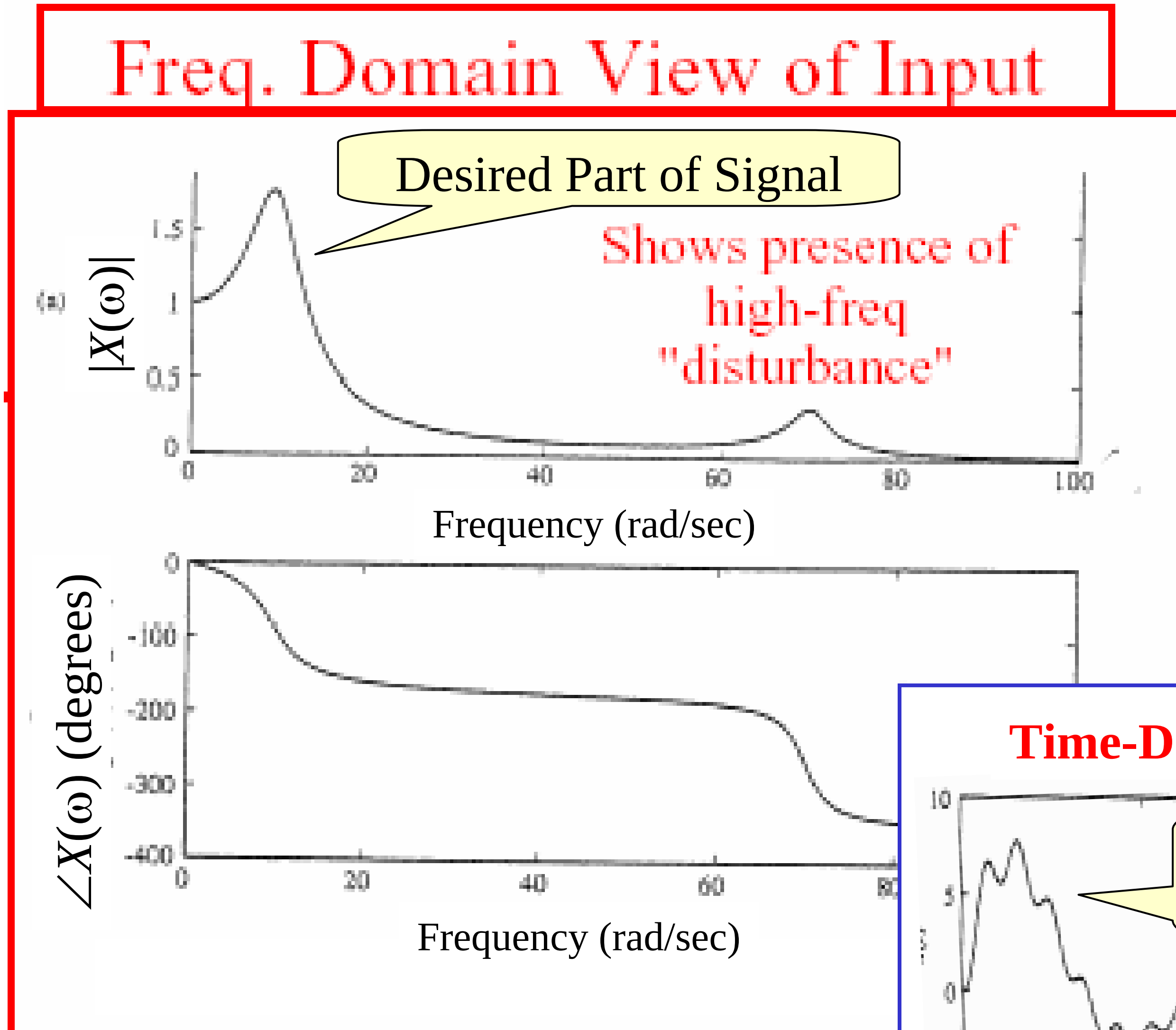
Alternate Input FT

Same Freq. Resp.

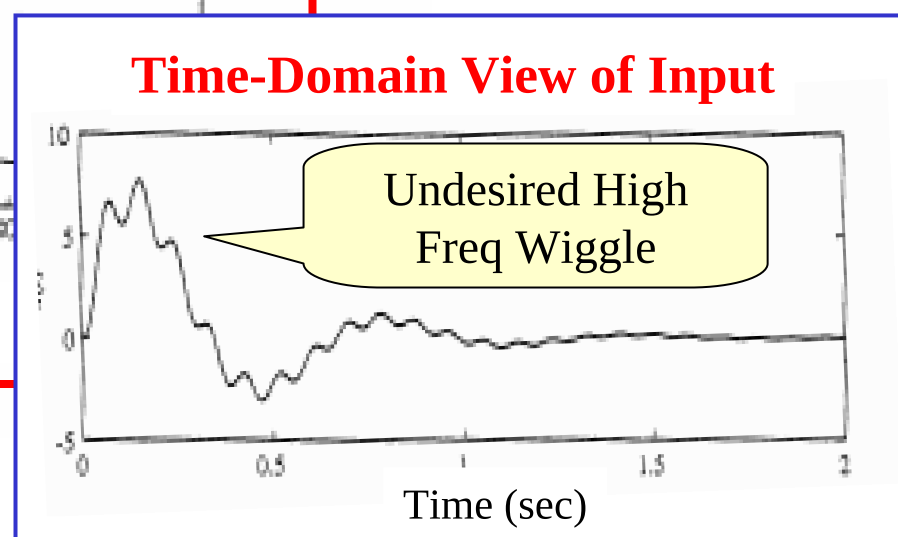
Alt. Output FT

Output FT is not changed much from Input FT: this is a viable pulse!!!

Example: Attenuation of high frequency Disturbance



This scenario could occur in an audio setting (a high-pitched interference). We've also seen it occur in the example of a radio receiver (the de-modulator created the desired low-freq signal but it also created undesired high-freq signals



Freq. Domain View of Input

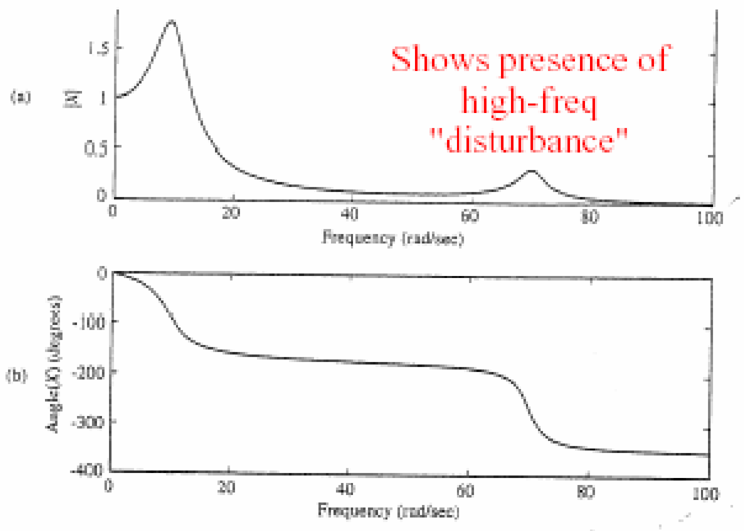


Figure 5.14 (a) Amplitude and (b) phase spectra of input in Example 5.6.

Freq. Domain View of System

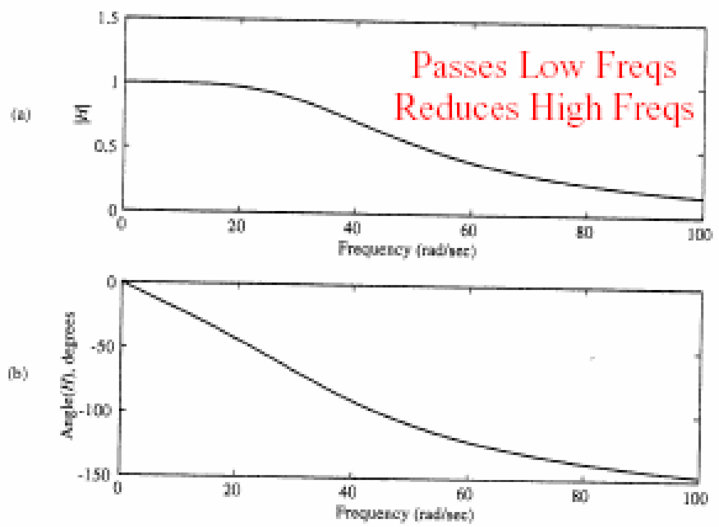


Figure 5.13 (a) Magnitude and (b) phase functions of system in Example 5.6.

Freq. Domain View of Output

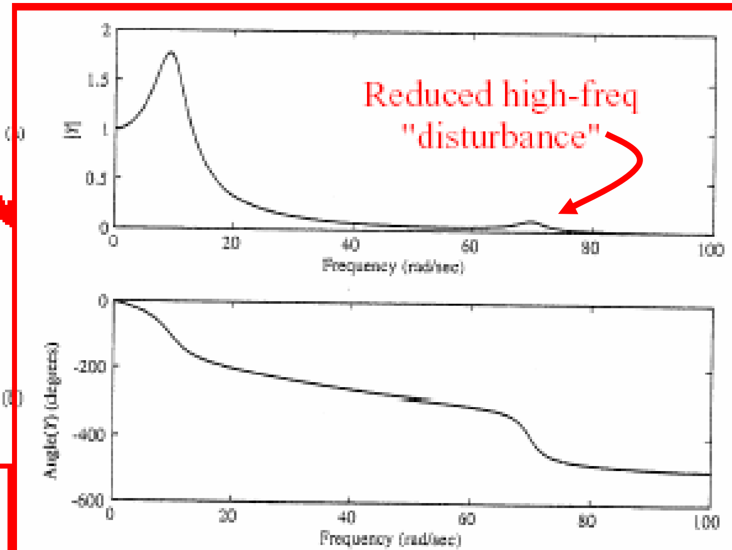
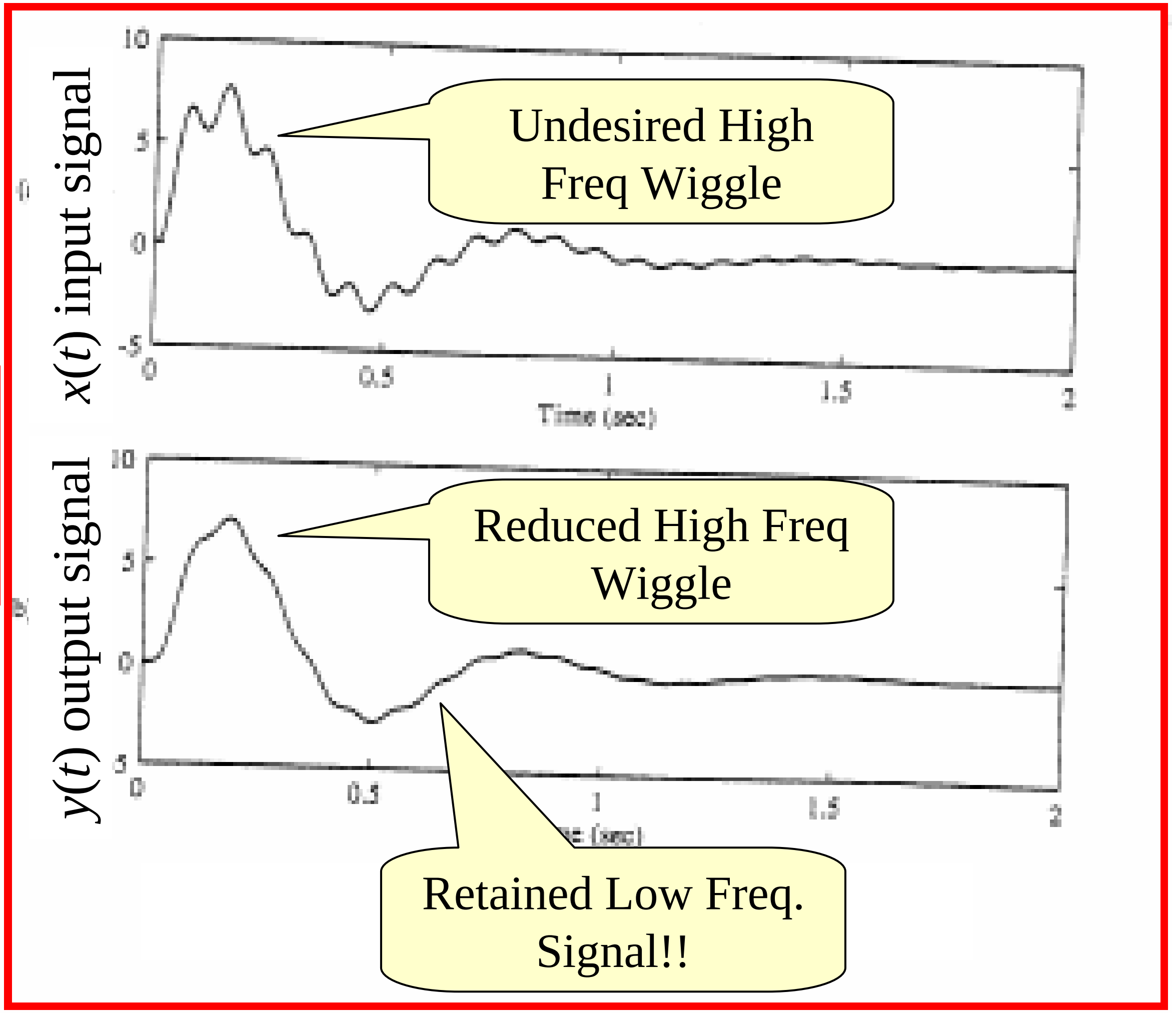


Figure 5.15 (a) Amplitude and (b) phase spectra of output in Example 5.6.

Time Domain View of Input & Output



Comments on This Example

- We can use the FT to "see" at what frequencies there are undesired signals
- Then we can specify a desired system frequency response $H(\omega)$ that will reduce (or "attenuate") the undesired signal while keeping the desired signal
  - Note that it would be virtually impossible to try to ***directly*** specify a desired system impulse response that will do this
- Once we have specified the desired $H(\omega)$ we could try to find a circuit (i.e., a physical system) that will implement it (either exactly or approximately)
  - This is the "design" or "system synthesis" problem
  - We haven't yet learned how to do this!! Tools we'll learn later will help!
  - However, if we have $H(\omega)$ specified as a mathematical function we could *possibly* compute the inverse FT to get the impulse response $h(t)$… then we could implement this "digitally" like we did earlier to simulate an RC circuit using D-T convolution.