BINGHAMTON UNIVERSITY

State University of New York

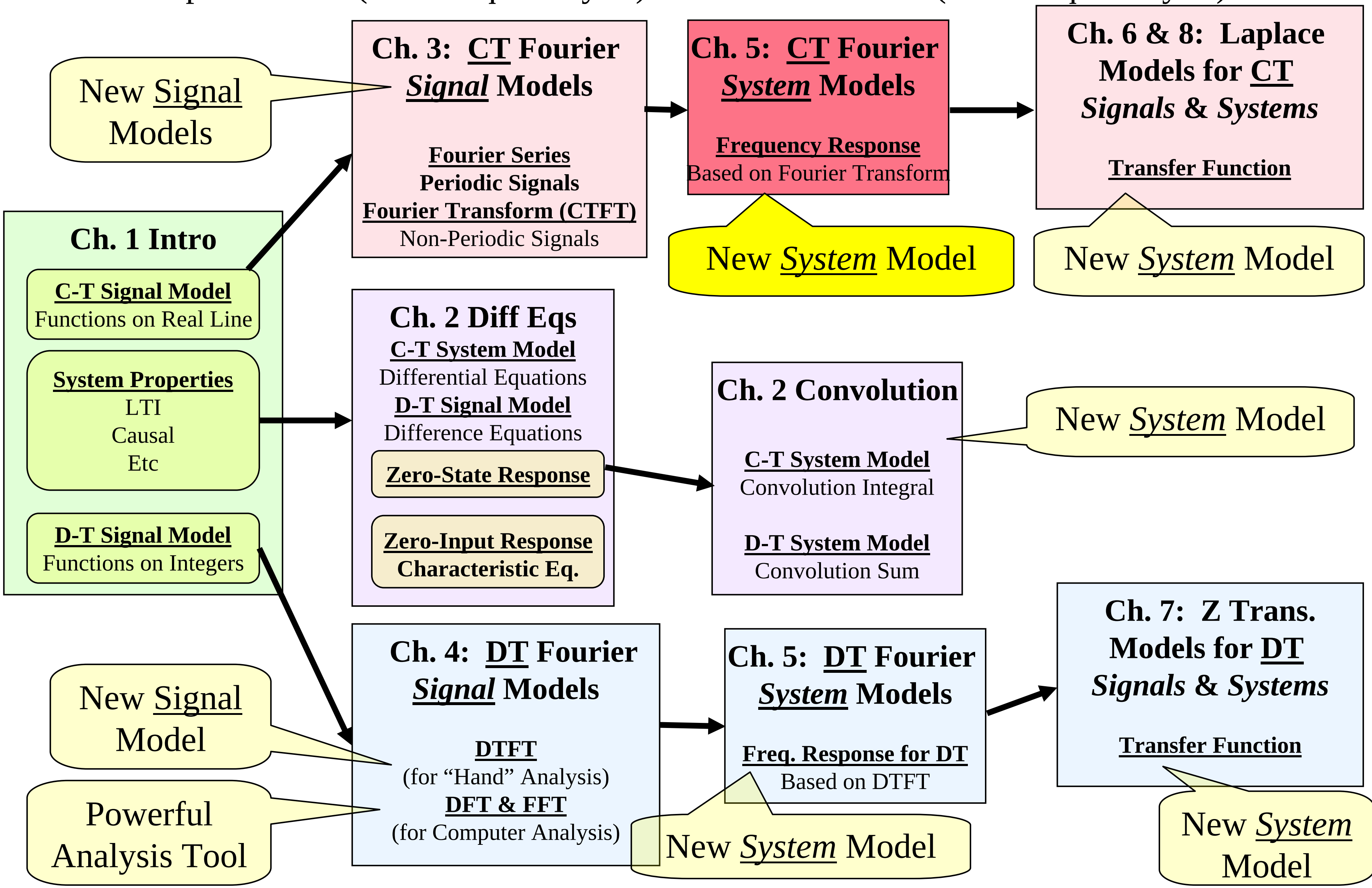
EECE 301
Signals & Systems
Prof. Mark Fowler

Note Set #20

- C-T Systems: Ideal Filters - Frequency-Domain Analysis
- Reading Assignment: Section 5.3 of Kamen and Heck

Course Flow Diagram

The arrows here show conceptual flow between ideas. Note the parallel structure between the pink blocks (C-T Freq. Analysis) and the blue blocks (D-T Freq. Analysis).



5.3 Ideal Filters

Often we have a scenario where we have a "good" signal, $x_g(t)$, corrupted by a "bad" signal, $x_b(t)$, and we want to use an LTI system to remove (or filter out) the bad signal, leaving only the good signal.

Desired Output

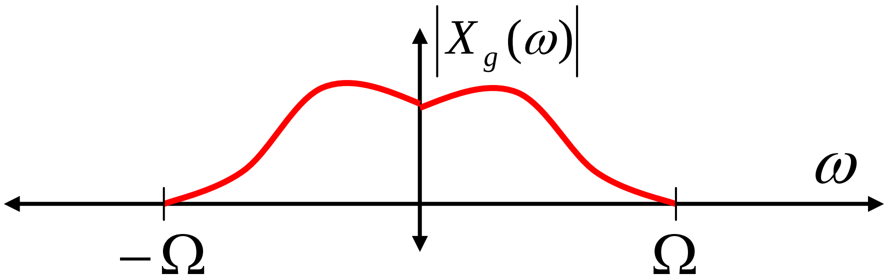
$$x(t) = x_g(t) + x_b(t) \longrightarrow \boxed{\begin{matrix} h(t) \\ H(\omega) \end{matrix}} \longrightarrow y(t) = x_g(t)$$

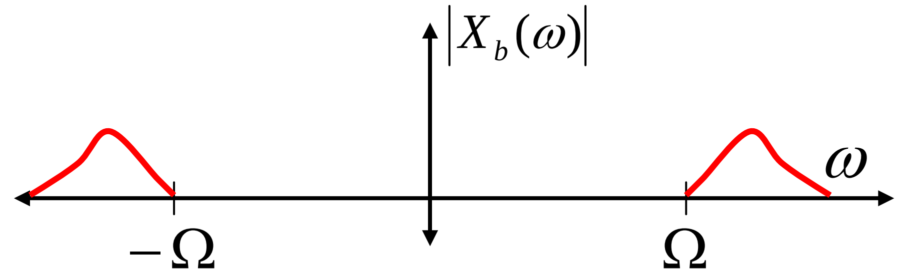
Called "a Filter"

How do we do this? What $H(\omega)$ do we want?

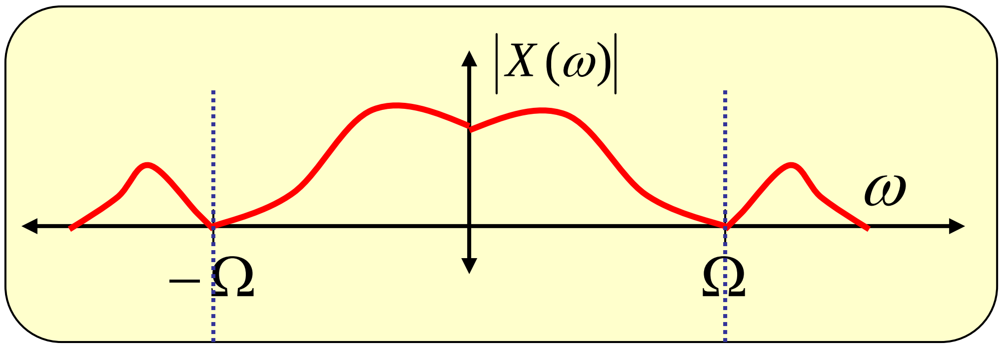
Note: You cannot design the circuit until you know which $H(\omega)$ the circuit must implement

Case #1: $x_g(t)$ is a low-frequency signal $x_b(t)$ is a high-frequency signal

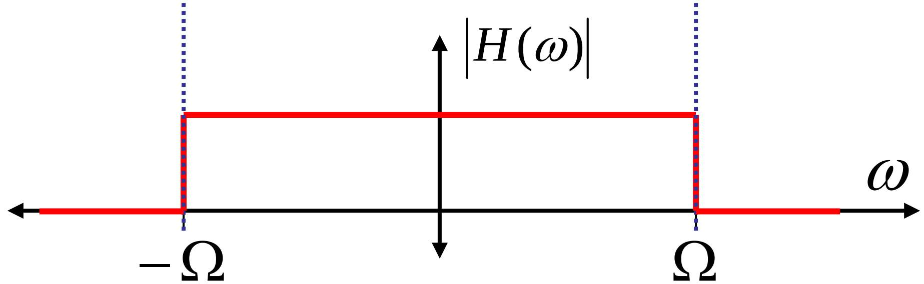




Spectrum of the Input Signal



In this case, we want a filter like this:



Mathematically:

$$\boxed{|H(\omega)| = \begin{cases} 1, & -\Omega < \omega < \Omega \\ 0, & otherwise \end{cases}}$$

"Passband"

"Stopband"

Then:

$$X(\omega) \longrightarrow \boxed{H(\omega)} \longrightarrow Y(\omega) = X(\omega)H(\omega)$$

$$|Y(\omega)| = \underbrace{|H(\omega)||X_g(\omega)|}_{=|X_g(\omega)|} + \underbrace{|H(\omega)||X_b(\omega)|}_{=0}$$

$$= |X_g(\omega)|$$

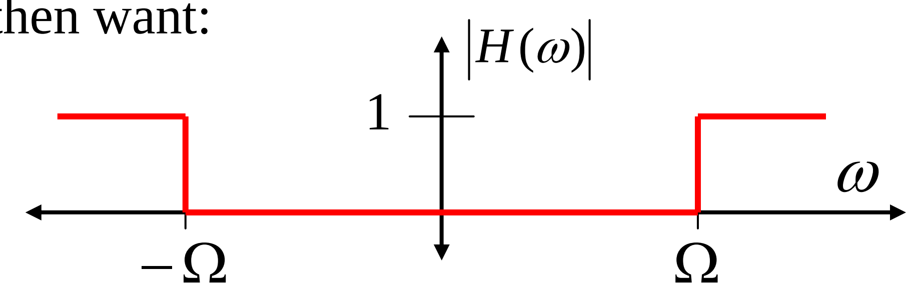
as desired

Such a filter is called a "low-pass filter"

Case #2: $X_g(\omega)$ is a high-frequency signal

$X_b(\omega)$ is a low-frequency signal

We then want:



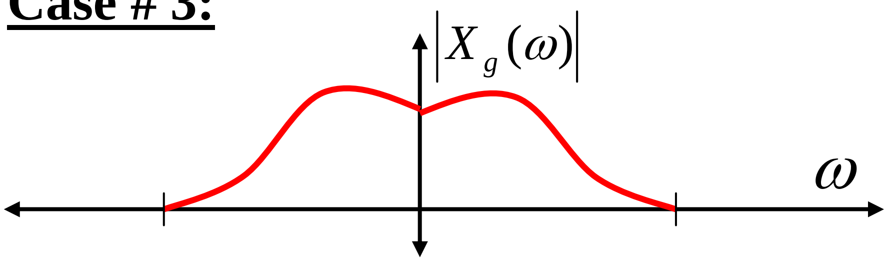
$$|H(\omega)| = \begin{cases} 0, & -\Omega < \omega < \Omega \\ 1, & otherwise \end{cases}$$

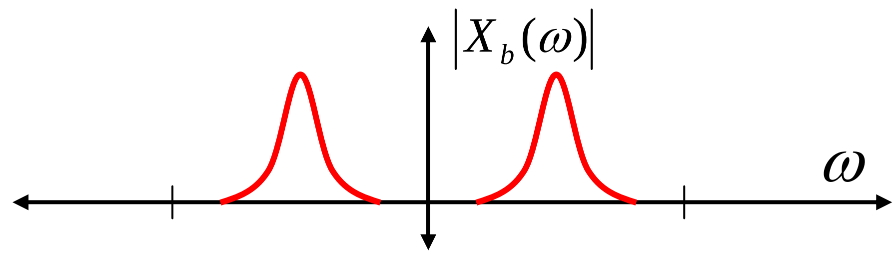
"Stopband"

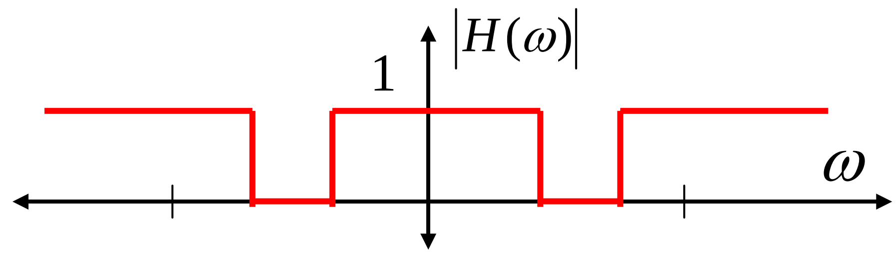
"Passband"

This is called a "high-pass filter"

Case # 3:

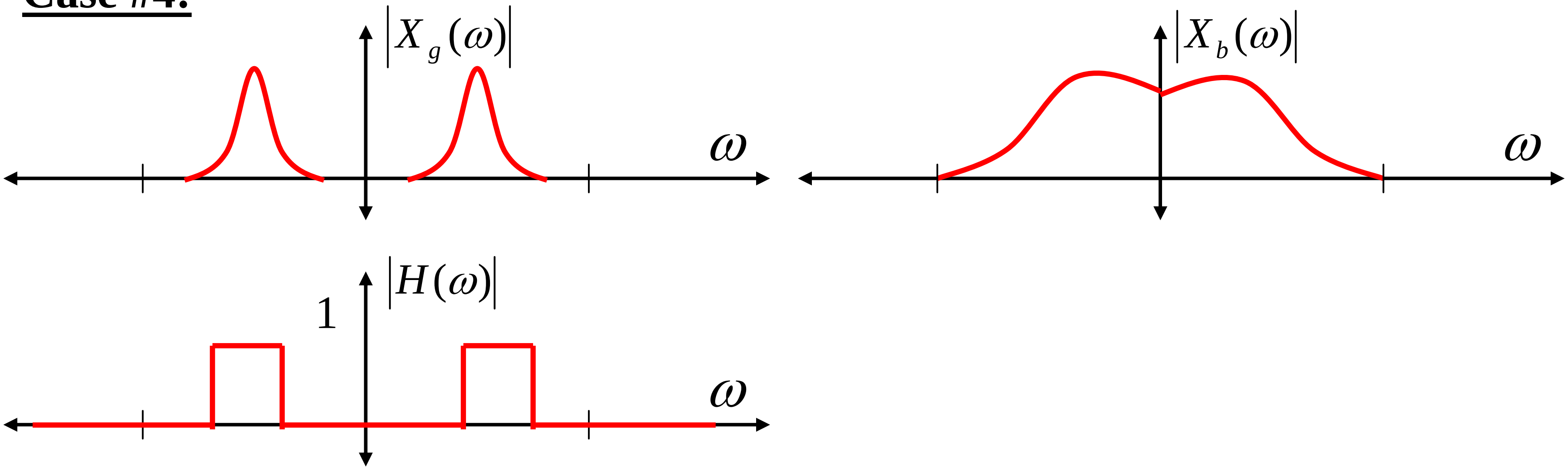






"Bandstop Filter" or "Notch Filter"

Case #4:



Bandpass Filter

Note that in Cases #3 and #4 the filter can't remove the bad signal without causing some damage to the desired signal…

…this is not specific to bandpass and bandstop filters…

…it can also happen with low-pass and high-pass filters.

In practice this is almost always the case!!

What about the ***phase*** of the filter's $H(\omega)$?

Well…we could tolerate a small delay in the output so…

Put in the signal we want "passed"

$$x_g(t) \longrightarrow \boxed{\begin{matrix} h(t) \\ H(\omega) \end{matrix}} \longrightarrow y(t) = x_g(t - t_d)$$

Want to get out the signal we want "passed"… but we can accept a "small" delay

From the time-shift property of the FT then we need:

$$\boxed{Y(\omega) = X_g(\omega)e^{-j\omega t_d}}$$

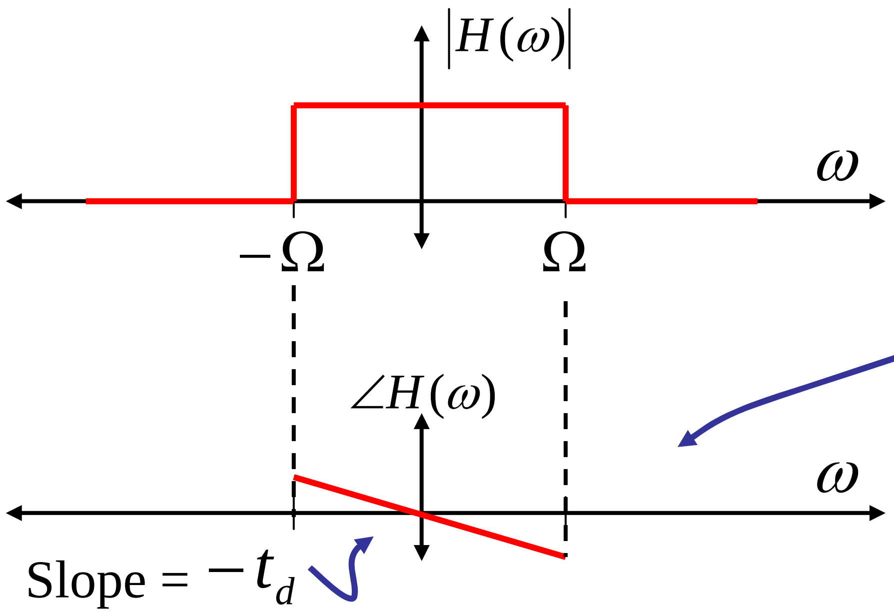
Thus we should treat the exponential term here as $H(\omega)$, so we have:

$$\boxed{\begin{aligned} |H(\omega)| &= \left|e^{-j\omega t_d}\right| = 1 \\ \angle H(\omega) &= \angle e^{-j\omega t_d} = -\omega t_d \end{aligned}}$$

For ω in the "pass band" of the filter

Line of slope $-t_d$ "Linear Phase"

So… for an ideal low-pass filter (LPF) we have:



$$H(\omega) = \begin{cases} 1e^{-j\omega t_d}, & -\Omega < \omega < \Omega \\ 0, & otherwise \end{cases}$$

Phase is undefined here:

$$0 = 0e^{j\theta}$$
$$\angle 0 = ?$$

i.e. phase is undefined for frequencies outside the ideal passband

Summary of *Ideal* Filters

1. Magnitude Response:
   a. Constant in Passband
   b. Zero in Stopband
2. Phase Response
   a. Linear in Passband (negative slope = delay)
   b. Undefined in Stopband

Example of the effect of a nonlinear phase but an ideal magnitude

Here is the scenario: Imagine we have a a signal $x(t)$ given by

$$x(t) = 9 - 5\cos(2\pi t) - 3\cos(2\pi 2t) - \cos(2\pi 3t)$$

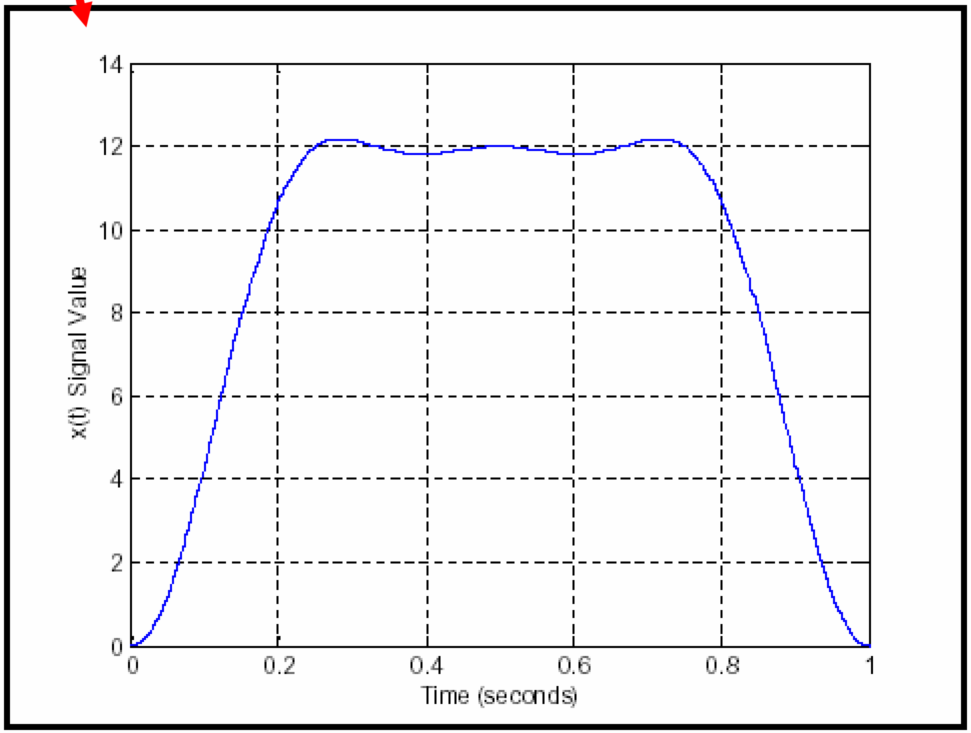
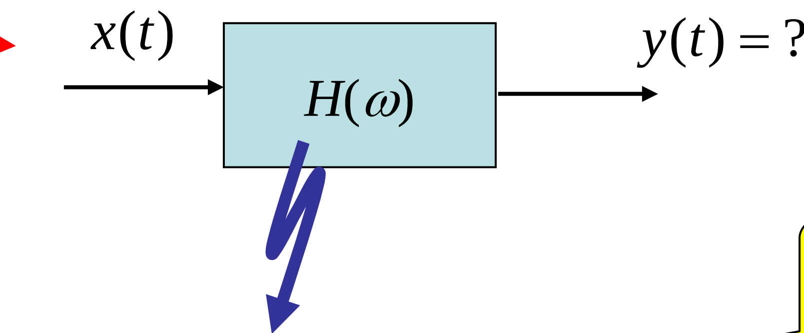


Figure 1: The input signal.

0 Hz 1 Hz 2 Hz 3 Hz

$$x(t) = 9 - 5\cos(2\pi t) - 3\cos(2\pi 2t) - \cos(2\pi 3t)$$



Circles Show the Frequencies in Input Signal

Filter does ***NOT*** change ***amplitudes*** of input components

Filter has ***Non***-Linear Phase

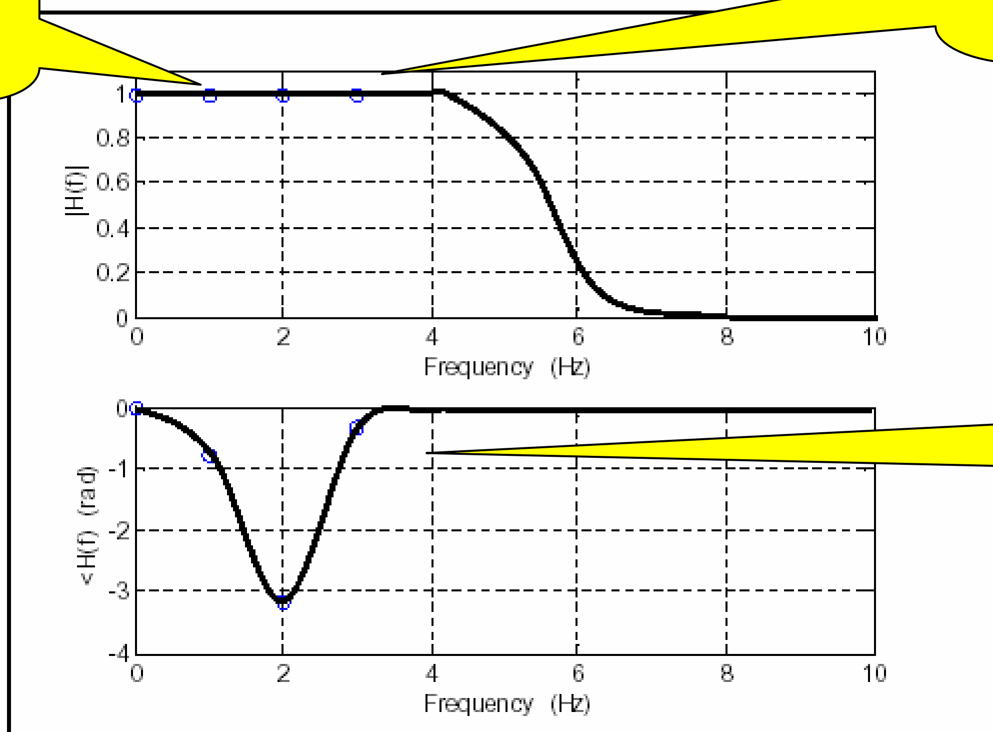
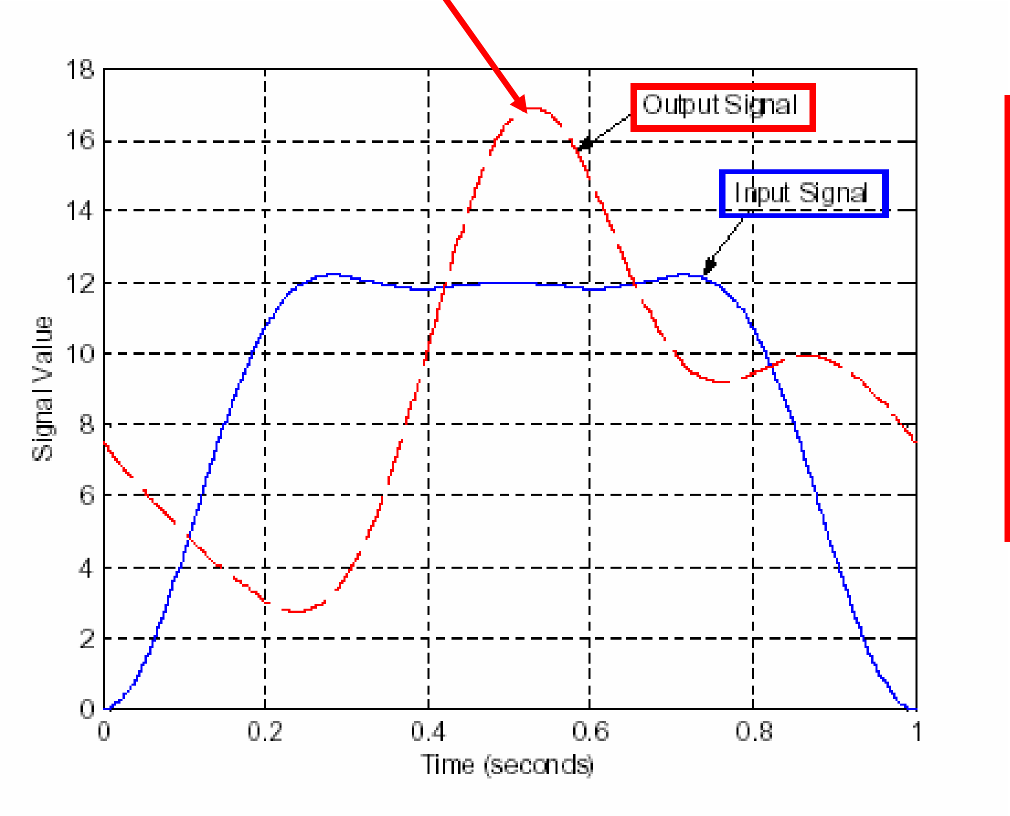


Figure 2: Filter's Frequency Response

So, at the filter's output we have four sinusoids at the same frequencies and amplitudes as at the input…BUT, they are not aligned in time in the same way they were at the input

$$y(t) = 9 - 5\cos(2\pi t - \frac{\pi}{4}) - 3\cos(2\pi 2t - \pi) - \cos(2\pi 3t - \frac{\pi}{10})$$



<u>Point of this Example</u>

A filter with an ideal magnitude response but non-ideal phase response can degrade a signal as much as a filter with a non-ideal magnitude response!!!