A

Example of LT Solution

First... A word about the 3 parts of the solution:

- Total Solution
  - Zero-Input Resp.
  - Zero-State Resp.
    - Transient Response
    - Steady-State Resp.

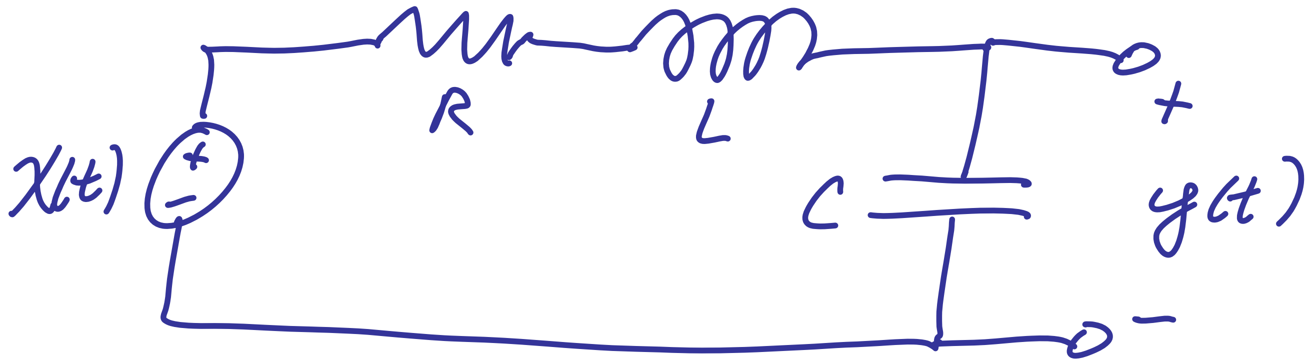
Zero-Input Resp.: This Decays (transient) if the system is stable. Its behaviour is set by the Char. Poly & the IC's

Den. of H(s)

Transient Response: This Decays (transient) if the system is stable. Its behaviour is set by the Char. Poly

Steady-State Resp.: This might not decay. Its behavior is set by the denominator of the Input X(s))

For this problem consider the RLC series circuit [B]



We've seen that the Transfer Function is:

$$H(s) = \frac{1/LC}{s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC}}$$

Let $L = 100\,mH$ $C = 8.5\,\mu F$ $R = 220\,\Omega$

$$\Rightarrow H(s) = \frac{B(s)}{A(s)} = \frac{1084.7^2}{s^2 + 220 s + 1084.7^2} = \frac{1084.7^2}{(s+916.9)(s+1283.1)}$$

(For this case we've seen that when $R > \frac{2L}{\sqrt{LC}}$ we get distinct real roots) c

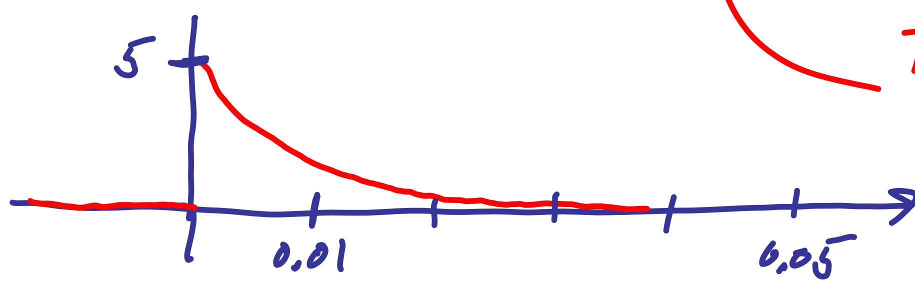
The TF $H(s)$ captures the structure of the system and gives us everything we need to solve for the circuit's output!

Suppose we have that:

<u>ICs</u>: $y(0^-) = 5 \qquad \dot{y}(0^-) = 2$

<u>Input</u>: $x(t) = 5e^{-100t}u(t)$

Time constant $= 1/100 = 0.01\,s$



From LT table: $\boxed{X(s) = \dfrac{5}{s+100}}$

We have seen – "Note Set 33 – CT used to solve Diff Eq." – that a 2^{nd}-order system has solution given by:

$$Y(s) = \underbrace{\frac{y(0^-)s + [\dot{y}(0^-) + a_1 y(0^-)]}{A(s)}}_{\text{Zero-Input Part}} + \underbrace{H(s)X(s)}_{\text{Zero-State Part}}$$

Now putting in for $A(s)$ & $H(s)$ gives:

$$Y(s) = \frac{y(0^-)s + [\dot{y}(0^-) + 2200\, y(0^-)]}{s^2 + 2200s + 1084.7^2} + \boxed{\left[\frac{1084.7^2}{s^2 + 2200s + 1084.7^2}\right]} X(s)$$

Note that $A(s)$ – the system's <u>char. Poly.</u> shows up here

Now plugging in the IC's gives: (E)

$$Y(s) = \frac{5s + 11002}{s^2 + 2200s + 1084.7^2} + \left[\frac{1084.7^2}{s^2 + 2200s + 1084.7^2}\right] X(s)$$

Now plugging in $X(s)$ gives:

$$Y(s) = \frac{5s + 11002}{s^2 + 2200s + 1084.7^2} + \left[\frac{1084.7^2}{s^2 + 2200s + 1084.7^2}\right]\left[\frac{5}{s+100}\right]$$

$$= \frac{5s + 11002}{s^2 + 2200s + 1084.7^2} + \frac{5.8824 \times 10^6}{s^3 + 2300s^2 + (1.3965 \times 10^6)s + (1.1765 \times 10^8)}$$

Zero-Input Part... has system char. Poly. $A(s)$ in Denominator

Zero-state Part... has combo of $A(s)$ & den of $X(s)$ in Denominator

Note: Can use "conv" command to compute this! Just believe me... or prove it yourself!

```
>> A=[1 (R/L) 1/(L*C)];
>> format long
>> Den=conv(A,[1 100])
Den =

  1.0e+008 *
   0.00000001000000   0.00002300000000   0.01396470588235   1.17647058823529
```

Now repeat last result here:

F

zero-Input — zero-State

$$Y(s) = \frac{5s + 11002}{s^2 + 2200s + 1084.7^2} + \frac{5.8824\times10^6}{s^3 + 2300s^2 + (1.3965\times10^6)s + (1.1795\times10^8)}$$

Do PFE on This — Do PFE on This

```
>>A=[1 (R/L) 1/(L*C)];
>>[R,P,K]=residue([5 11002],A)
R =
  -12.5237
   17.5237
P =
  1.0e+003 *
   -1.2831
   -0.9169
```

```
>>A=[1 (R/L) 1/(L*C)];
>>Den=conv([1 100],A);
>>[R,P,K]=residue(5*(1/(L*C)),Den)
R =
   13.5763
  -19.6628
    6.0864
P =
  1.0e+003 *
   -1.2831
   -0.9169
   -0.1000
```

same

These are roots of char. Poly. A(s)!!

root of s+100 — the den. of X(s)!!

$$Y(s) = \left[\frac{-12.52}{s+1283.1} + \frac{17.52}{s+916.9}\right] + \left[\left[\frac{13.58}{s+1283.1} + \frac{-19.66}{s+916.9}\right] + \frac{6.09}{s+100}\right]$$

(G)

$$Y(s) = \left[\frac{-12.52}{s+1283.1} + \frac{17.52}{s+916.9}\right] + \left[\left[\frac{13.58}{s+1283.1} + \frac{-19.66}{s+916.9}\right] + \frac{6.09}{s+100}\right]$$

Factors of Char. Poly A(s) | Factors of Char. Poly A(s) | Den. of X(s)

Zero-Input Part

Zero-State Part

General Rules!!

- Transient Due to Input — is the part of ZS Resp. that is due to the factors of A(s)
- Steady-State Resp to Input — is the part of ZS Resp. that is due to the factors of the Den. of X(s)

Now use the LT table to invert: (11)

$$Y(s) = \left[\frac{-12.52}{s+1283.1} + \frac{17.52}{s+916.9}\right] + \left[\left[\frac{13.58}{s+1283.1} + \frac{-19.66}{s+916.9}\right] + \frac{6.09}{s+100}\right]$$

$$\boxed{\begin{aligned} y(t) &= -12.52e^{-1283.1t}u(t) + 17.52e^{-916.9t}u(t) \leftarrow \text{ZI Part} \\ &+13.58e^{-1283.1t}u(t) - 19.66e^{-916.9t}u(t) \leftarrow \text{ZS Trans. Part} \\ &+6.09e^{-100t}u(t) \leftarrow \text{ZS steady state Part} \end{aligned}}$$

For this example the ZI part & ZS Trans. Part die out long before the steady state Part decays away.