BINGHAMTON
UNIVERSITY
State University of New York

EECE 301
Signals & Systems

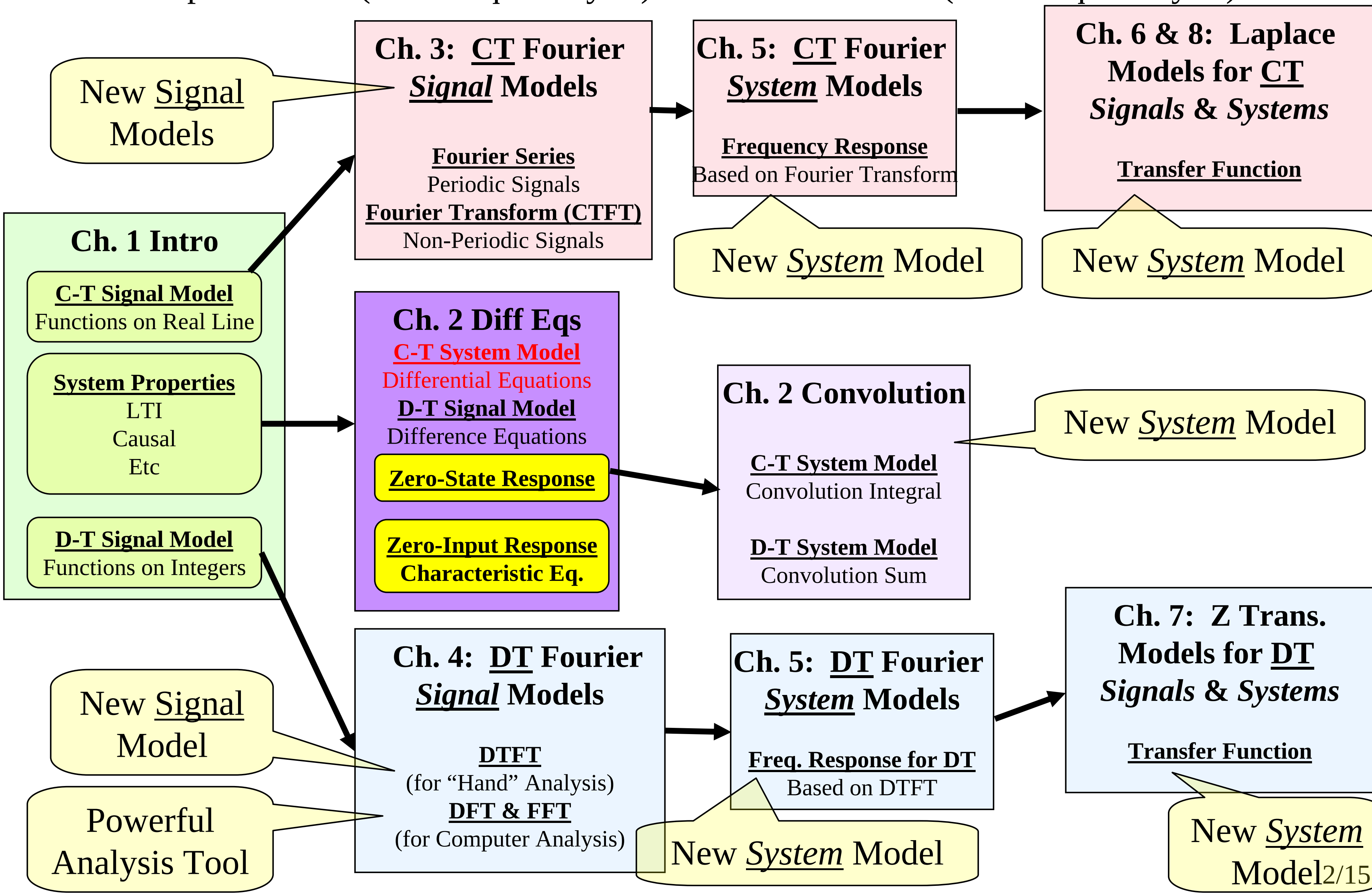
Prof. Mark Fowler

Note Set #6

- System Modeling and C-T System Models
- Reading Assignment: Sections 2.4 & 2.5 of Kamen and Heck

Course Flow Diagram

The arrows here show conceptual flow between ideas. Note the parallel structure between the pink blocks (C-T Freq. Analysis) and the blue blocks (D-T Freq. Analysis).



System Modeling

To do engineering design, we must be able to accurately predict the quantitative behavior of a circuit or other system.

This requires math models:

Circuits

- Device Rules
  - R: $v(t)=Ri(t)$
  - L: $v(t)=L(di(t)/di)$
  - C: $dv(t)/dt=1/Ci(t)$
- Circuit Rules
  - -KVL
  - -KCL
  - -Voltage Divider
  - -etc.

→ **Differential Equation**

Mechanical

- Device Rules
  - Mass: $M(d^2p(t)/dt^2)$
  - Spring: $k_x p(t)$
  - Damping: $k_d(dp(t)/dt)$
- System Rules
  - -Sum of forces
  - -etc.

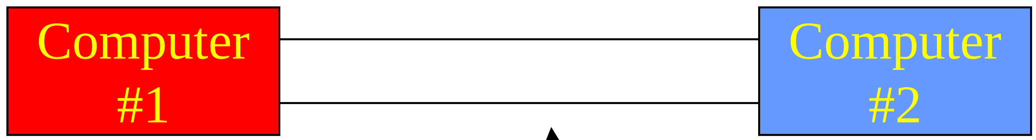
→ **Differential Equation**

Similar ideas hold for hydraulic, chemical, etc. systems…

"differential equations rule the world"

Simple Circuit Example:

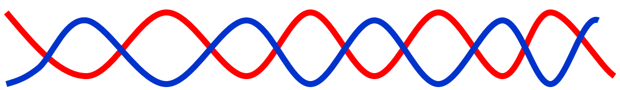
Sending info over a wire cable between two computers



Two conductors separated by an insulator
$\Rightarrow$ capacitance

Two practical examples of the cable

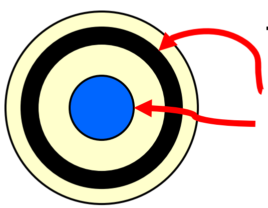
“Twisted Pair” of Insulated Wires



Typical values: 100 Ω/km

50 nF/km

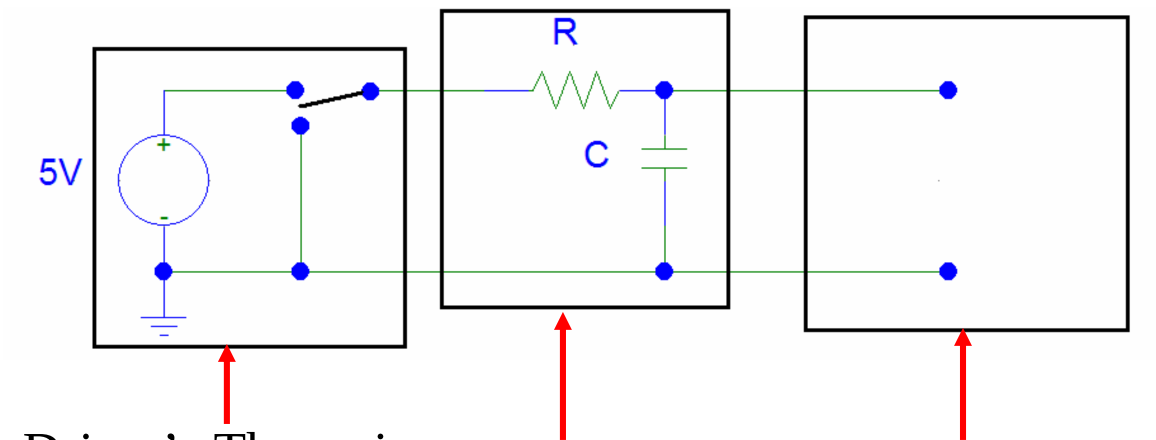
coaxial cable



conductors separated by insulator

Recall: resistance increases with wire length

Simple Model:



Driver's Thevenin Equivalent Circuit (Computer #1)

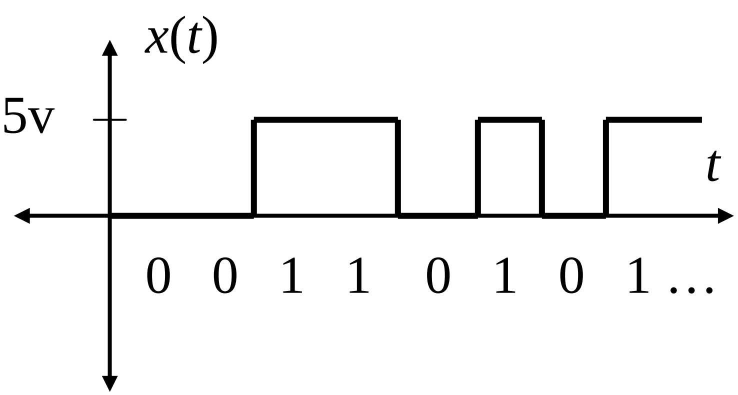
Cable Model

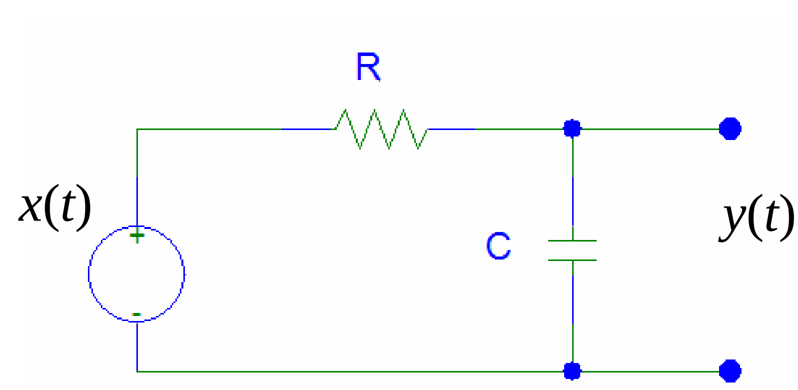
Receiver's Thevenin Equivalent Circuit (Computer #2)

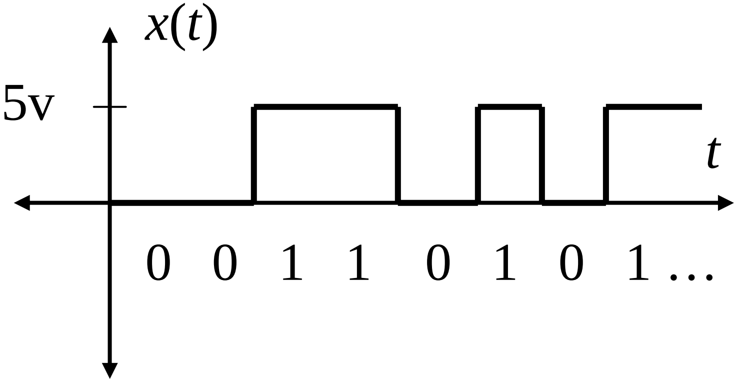
Zero Output Resistance (Ideal)

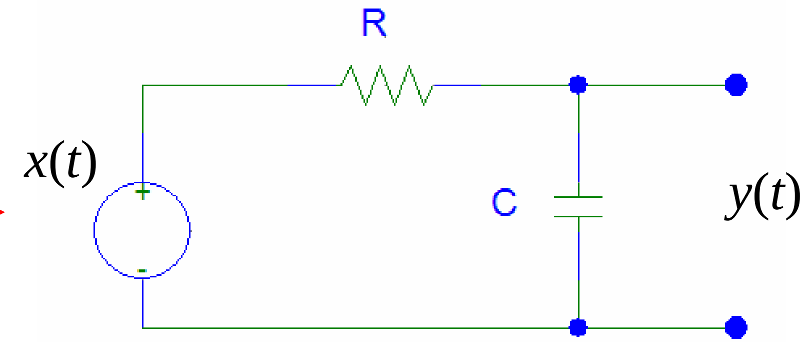
Infinite Input Resistance (Ideal)

Effective Operation:









Use Loop Equation & Device Rules:

$$x(t) = v_R(t) + y(t)$$

$$v_R(t) = Ri(t)$$

$$i(t) = C\frac{dy(t)}{dt}$$

$$\frac{dy(t)}{dt} + \frac{1}{RC}y(t) = \frac{1}{RC}x(t)$$

This is the Differential Equation to be "Solved":

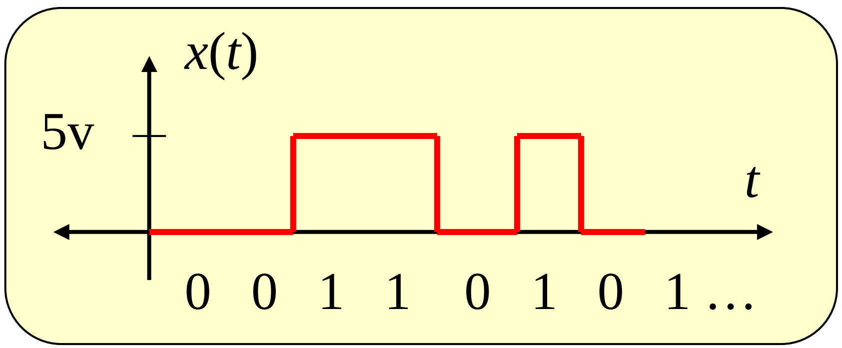
Given: Input $x(t)$ Find: Solution $y(t)$

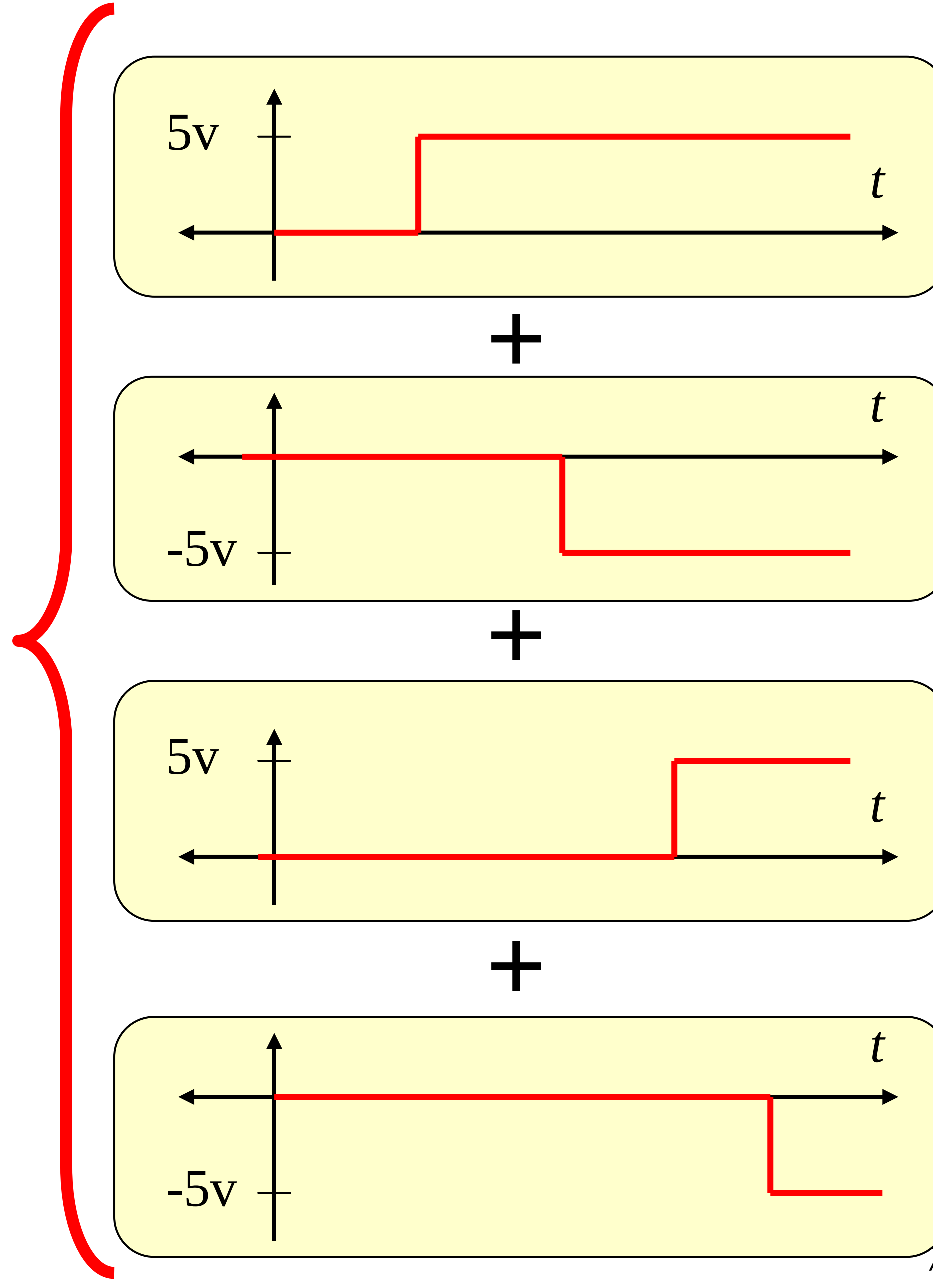
Recall: A "Solution" of the D.E. means… The function that when put into the left side causes it to reduce to the right side

Differential Equation & System
… the solution is the output

Now… because this is a **linear** system (it only has *R*, *L*, *C* components!) we can analyze it by **superposition**.

Decompose the input…

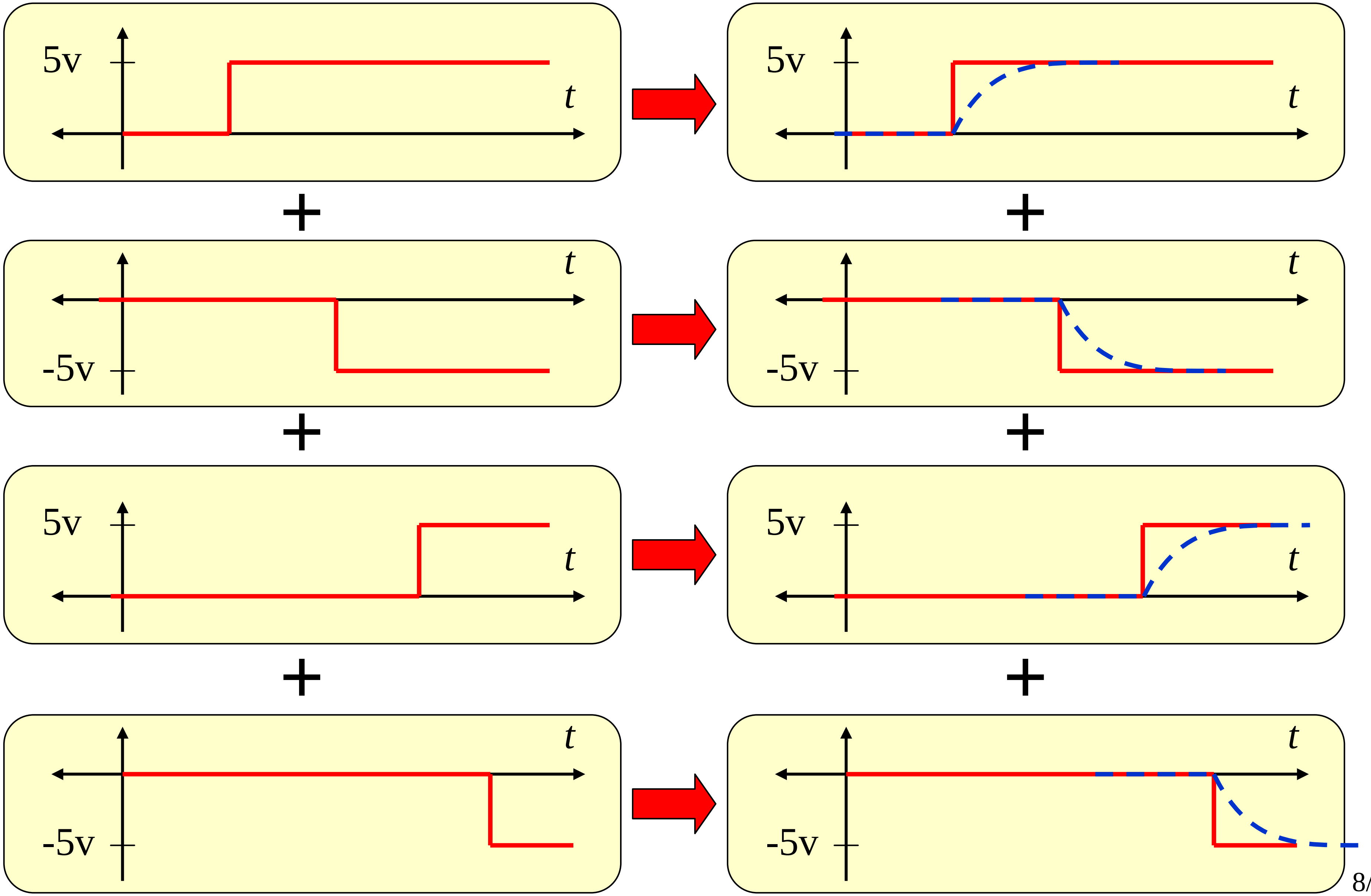




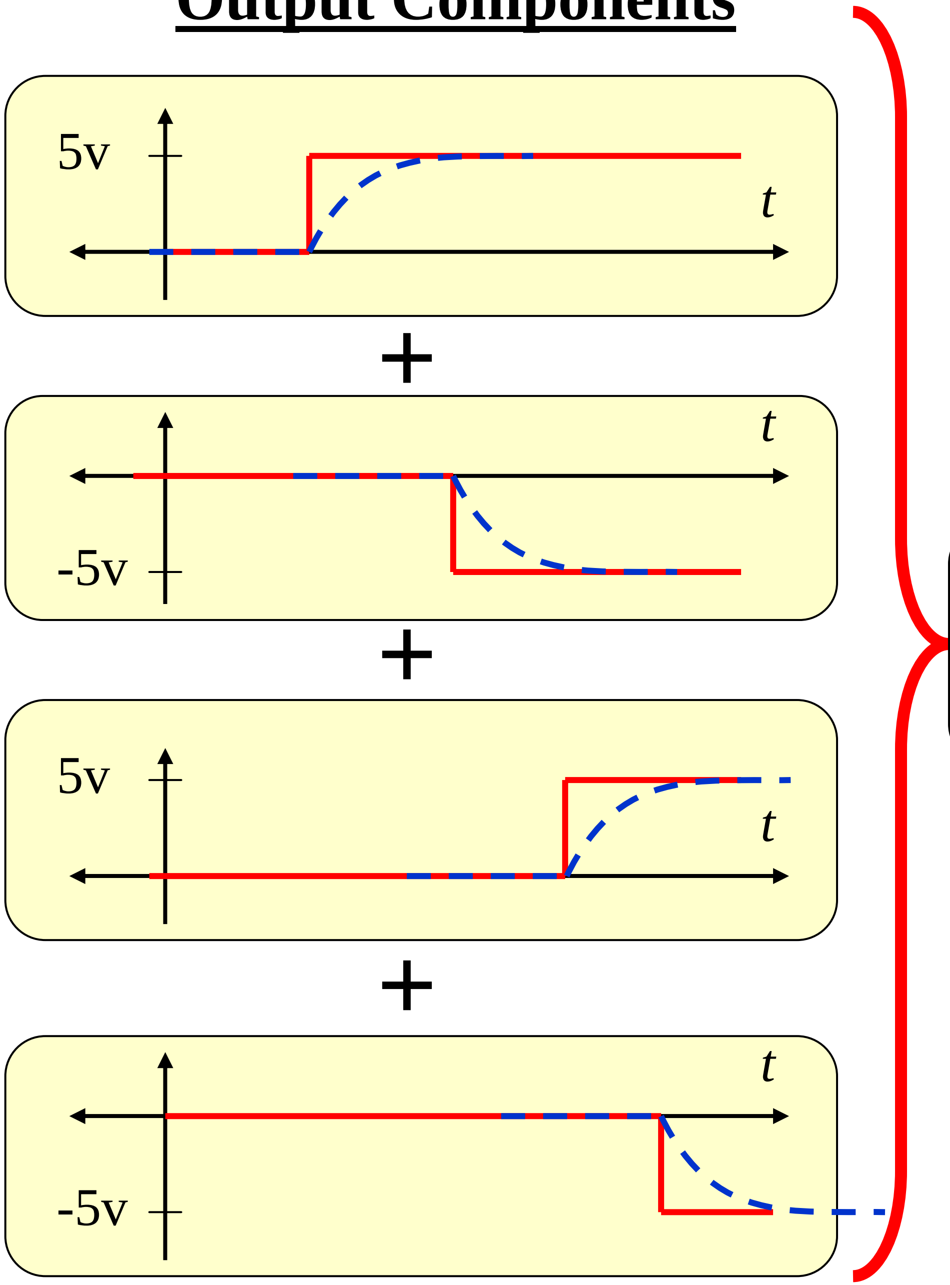
Input Components

Output Components (Blue)

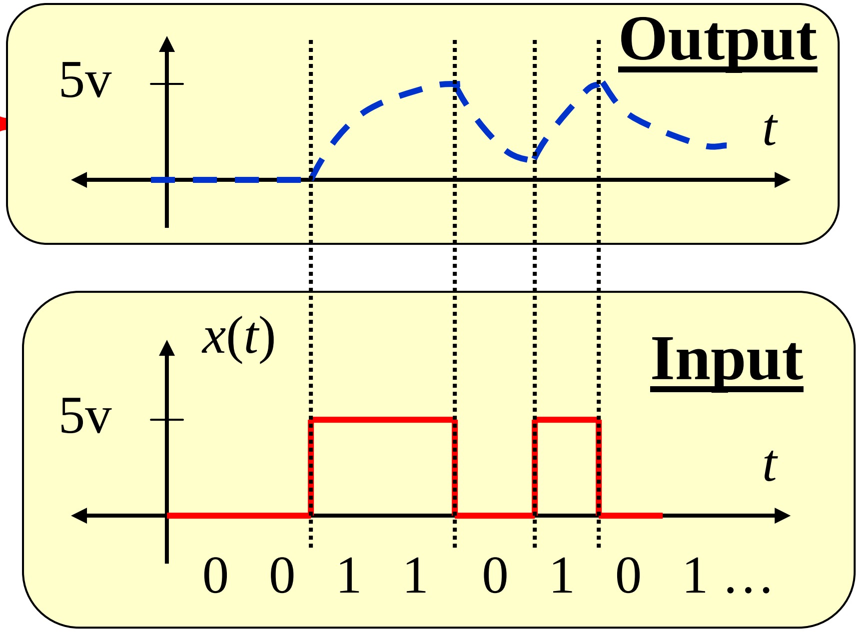
Standard Exponential Response
Learned in “Circuits”:



Output Components

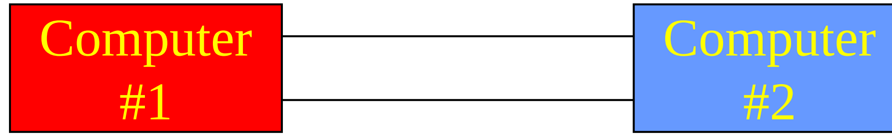


Output is a “smoothed” version of the input… it is harder to distinguish “ones” and “zeros”… it will be even harder if there is noise added onto the signal!

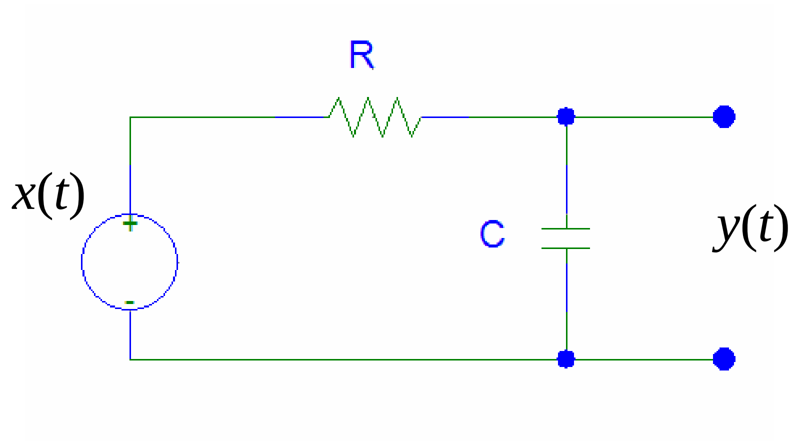


Progression of Ideas an Engineer Might Use for this Problem

Physical System:



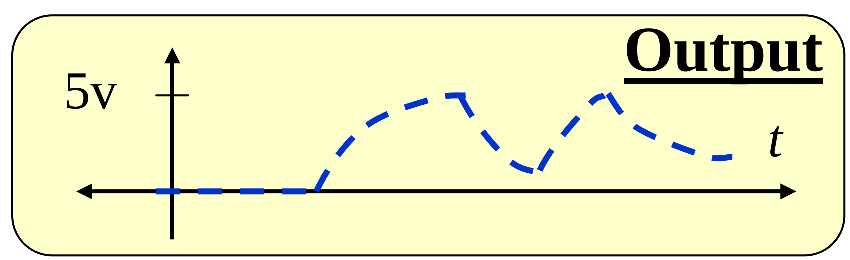
Schematic System:



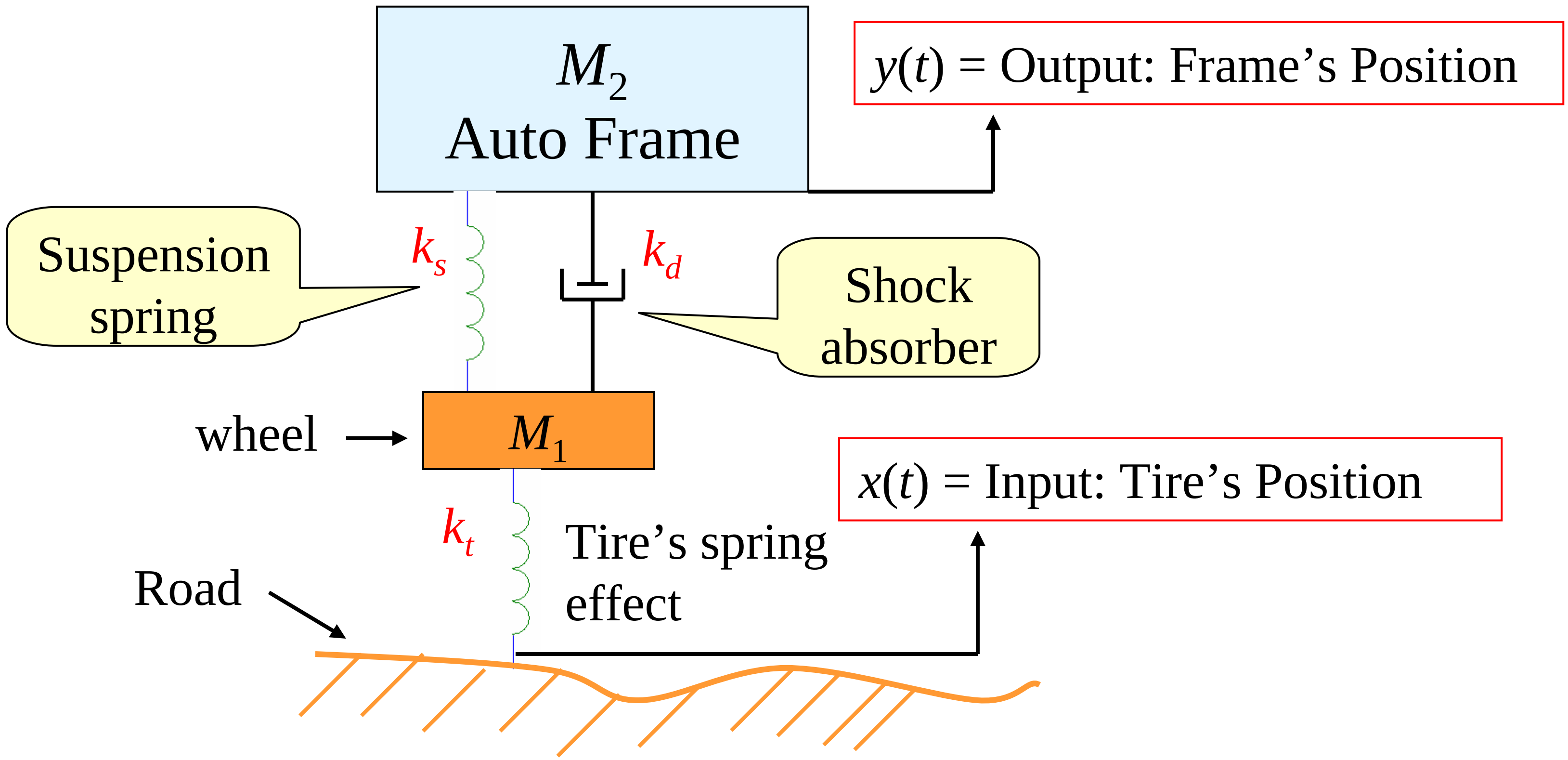
Mathematical System:

$$\frac{dy(t)}{dt} + \frac{1}{RC} y(t) = \frac{1}{RC} x(t)$$

Mathematical Solution:



Automobile Suspension System Example



Results in 4th order differential equation:

$$\frac{d^4 y(t)}{dt^4} + \frac{a_3 d^3 y(t)}{dt^3} + \frac{a_2 d^2 y(t)}{dt^2} + \frac{a_1 dy(t)}{dt} + a_0 y(t) = \underbrace{F[x(t)]}$$

Some function of Input $x(t)$

The a_i are functions of system's physical parameters:

M_1, M_2, k_s, k_d, k_t

Again… to find the output for a given input requires solving the differential equation

Engineers could use this differential equation *model* to theoretically explore:

1. How the car will respond to some typical theoretical test inputs when different possible values of system physical parameters are used
2. Determine what the best set of system physical parameters are for a desired response
3. Then… maybe build a prototype and use it to fine tune the real-world effects that are not captured by this differential equation model

So… What we are seeing is that for an engineer to analyze or design a circuit (or a general physical system) there is almost always an underlying Differential Equation whose solution for a given input tells how the system output behaves

So… engineers need both a qualitative and quantitative understanding of Differential Equations.

The major goal of this course is to provide tools that help gain that qualitative and quantitative understanding!!!

Linear Constant-Coefficient Differential Equations

General Form: (N^{th} - order)

$$y^{(N)}(t) + \sum_{i=0}^{N-1} a_i y^{(i)}(t) = \sum_{i=0}^{M} b_i x^{(i)}(t)$$

Indicates i^{th} order derivative

Input: $x(t)$

Output: $y(t)$ — Solution of the Differential Equation

Recall: Two parts to the solution

(i) one part due to ICs with zero-input ("zero-input response") — "Homogeneous Solution"

(ii) one part due to input with zero ICs ("zero-state response")

Characteristic Polynomial: $\lambda^N + a_{n-1}\lambda^{N-1} + \ldots + a_1\lambda + a_0$

N roots: $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_N$

N "modes": Assuming distinct roots… $e^{\lambda_1 t}, e^{\lambda_2 t}, \ldots, e^{\lambda_N t}$

$$\Rightarrow \quad y_{ZI}(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + \cdots + C_N e^{\lambda_N t}$$

See Video Review

Then: $y(t) = y_{ZI}(t) + y_{ZS}(t)$ ($y_{ZS}(t)$ is our focus, so we will often say ICs = 0)

So how do we find $y_{ZS}(t)$?

If you examine the zero-state part for all the example solutions of differential equations we have seen you'll see that they all look like this:

$$y_{ZS}(t) = \int_{t_0}^{t} h(t-\lambda)x(\lambda)d\lambda$$

Input

Output when "in zero state"

This is called "**Convolution**" (We'll study it in Ch. 2)

So we need to find out:

1. Given a differential equation, what is $h(t-\lambda)$

Really just need to know $h(t)$… it is called the system's "**Impulse Response**"

See Ch. 3, 5, 6, 8

2. How do we compute & understand the convolution integral

See Ch. 2

3. Are there other (easier? more insightful?) methods to find $y_{ZS}(t)$

See Ch. 3, 5, 6, 8