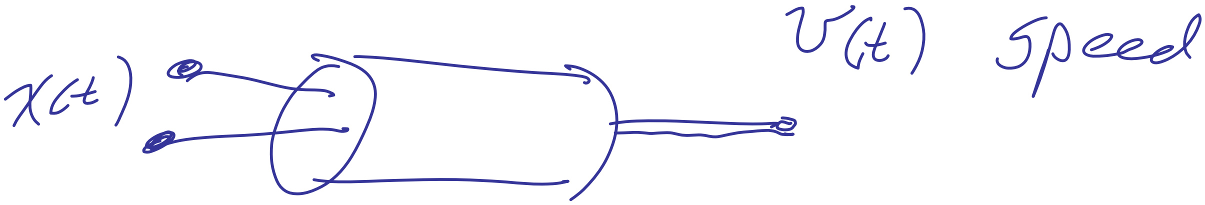
Example of LT Analysis



DC Motor

Recall from Discussion #3 we found the Diff. Eq. for the speed of a DC motor with an applied voltage $x(t)$:

$$\frac{d^2 v(t)}{dt^2} + \left(\frac{R}{L} + \frac{b}{I}\right)\frac{dv(t)}{dt} + \left(\frac{K_e K_t + Rb}{LI}\right) v(t) = \frac{K_t}{LI} x(t)$$

This can be written as:

$$\frac{d^2 v(t)}{dt^2} + a_1 \frac{dv(t)}{dt} + a_0 v(t) = b_0 x(t)$$

These depend on motor parameters

Now... Assume the motor is not turning (zero IC's) & take LT:

B

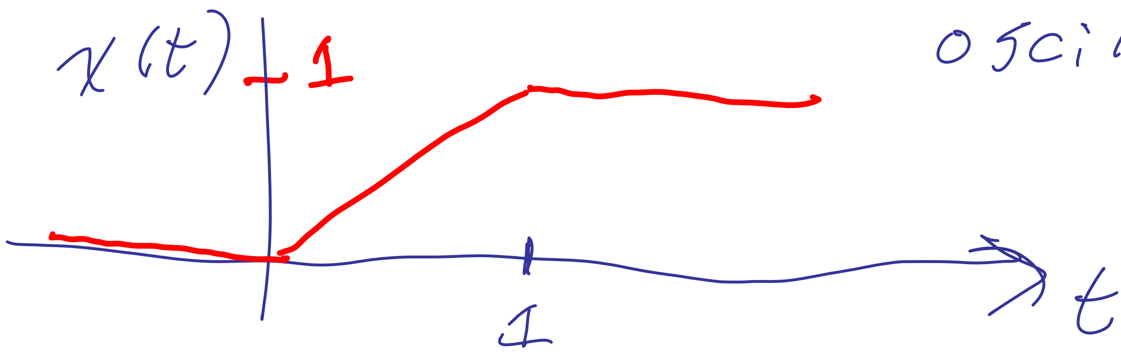
$$H(s) = \frac{b_0}{s^2 + a_1 s + a_0}$$

Assume that the motor parameters evaluate to

$$H(s) = \frac{18}{s^2 + 0.6s + 9}$$ → complex roots

Consider:

⇒ expect oscillation!



Intent: Linearly accelerate the motor up to speed then hold steady speed!

$$x(t) = t\,u(t) - (t-1)\,u(t-1)$$

C

From LT Tables:

$$X(s) = \frac{1}{s^2} - \frac{1}{s^2}e^{-s}$$

$$Y(s) = H(s)X(s)$$

$$= \frac{H(s)}{s^2} - \underbrace{\frac{H(s)}{s^2}}_{\text{Delay of this}} e^{-s}$$

Find $\mathcal{L}^{-1}\left\{\frac{H(s)}{s^2}\right\}$

$$\frac{H(s)}{s^2} = \frac{18}{s^2(\underbrace{s^2+0.6s+9}_{s^2+2\omega_n \zeta s+\omega_n^2})}$$

$\zeta = 0.1$

"Damping"

$$= \frac{18}{s^4+0.6s^3+9s^2}$$

Do Partial Fractions via MATLAB:

$$\frac{H(s)}{s^2} = \frac{-0.13}{s} + \frac{2}{s^2} + \frac{0.0667 + j0.33}{s + 0.3 - j2.99} + \frac{0.0667 - j0.33}{s + 0.3 + j2.99}$$

$$\downarrow \mathcal{L}^{-1}\{\ \}$$

Euler!

$$y_1(t) = -0.13\,u(t) + 2t\,u(t) + \left(0.34 e^{+j1.37}\right) e^{-0.3t} e^{+j2.99t} + \left(0.34 e^{-j1.37}\right) e^{-0.3t} e^{-j2.99t}$$

$$= -0.13u(t) + 2tu(t) + 0.34\, e^{-0.3t} \cos(2.99t + 1.37)$$

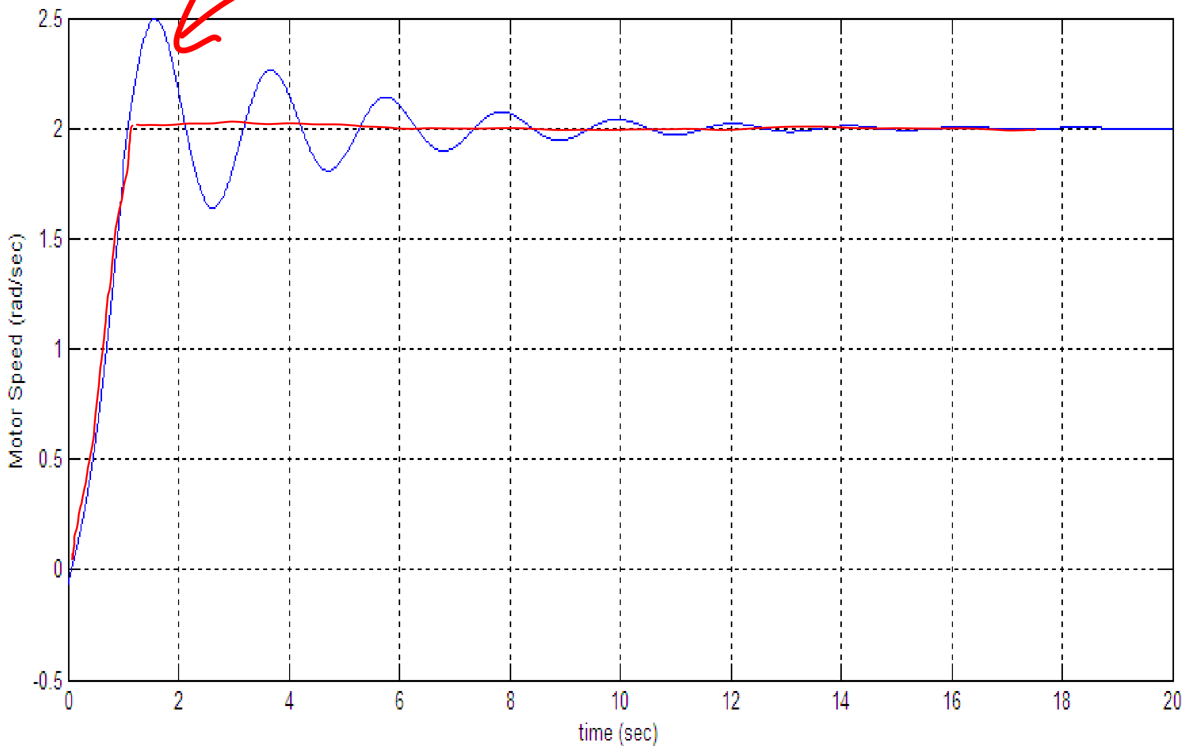
The other part is just a delay:

$$y_2(t) = y_1(t-1)$$

$$\text{\&} \quad y(t) = y_1(t) - y_1(t-1)$$

The response looks like this:

"Overshoot"



Now, the overshoot occurs because we' the ζ is too low ($\zeta = 0.1$)

The parameter "b" provides more damping:

$$\frac{d^2 v(t)}{dt^2} + \left(\frac{R}{L} + \frac{b}{I}\right)\frac{dv(t)}{dt} + \left(\frac{K_e K_t + Rb}{LI}\right) v(t) = \frac{K_t}{LI} x(t)$$

Suppose we increased ζ to give

$$H(s) = \frac{18}{s^2 + 7s + 9}$$

was 0.6, now is 7

The a similar analysis leads to:

"LAG" or "sluggishness"