Binghamton University
State University of New York

EECE 301
Signals & Systems
Prof. Mark Fowler

Note Set #18

- D-T Signals: Frequency-Domain Analysis

<u>Fourier Analysis of D-T Signals</u>

We now develop "Fourier ideas" for D-T signals like we did for C-T signals:

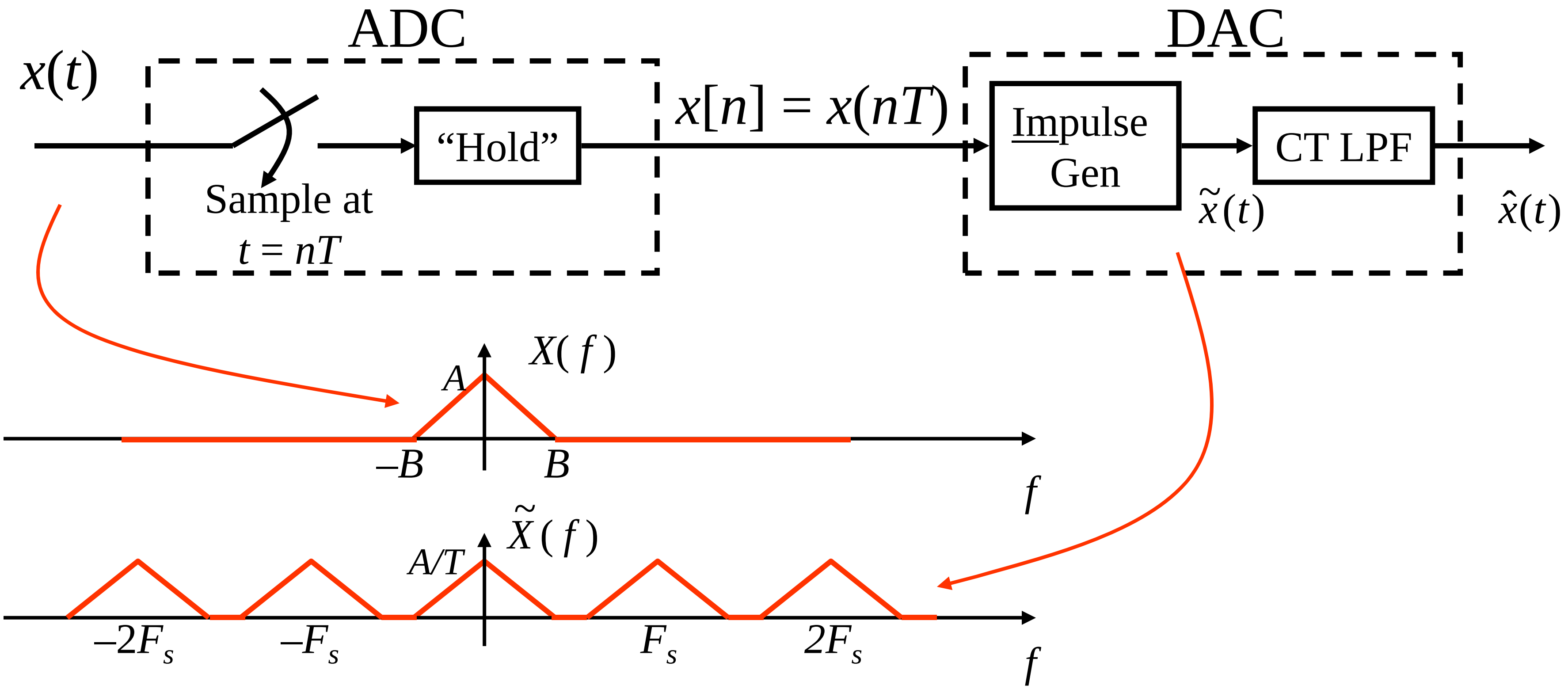
– Define a D-T FT (DTFT) for D-T signals and see that it works pretty much like the FT for C-T signals (CTFT)

But… we also do something we can't do for CTFT-based ideas:

– Develop a <u>computer-processing version</u> of the DTFT… called the Discrete Fourier Transform (DFT) that will allow you to use the computer to numerically compute a "view" of the DTFT

– But to make this DFT useful we'll need to understand the relationships between the DFT, the DTFT, and the CTFT!

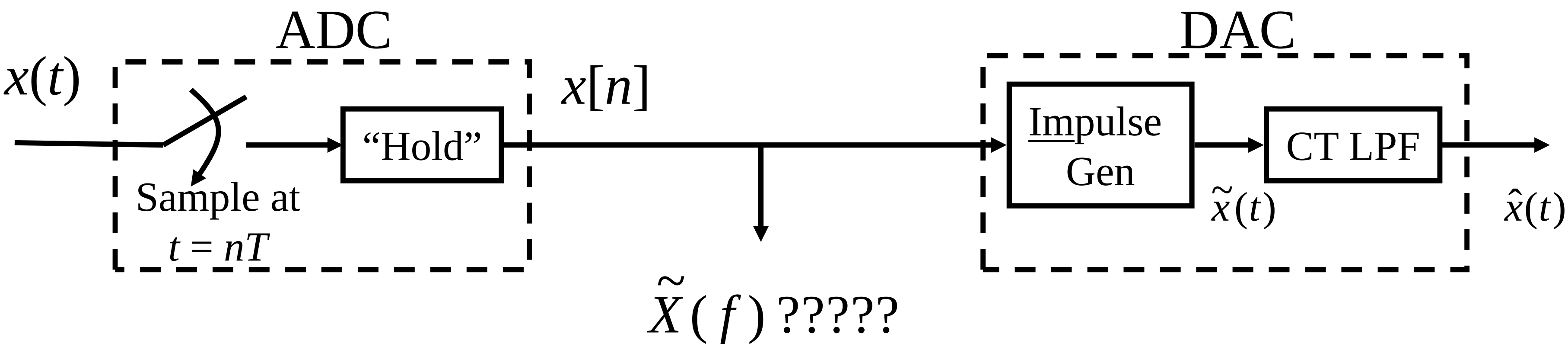
Recall: Sampling Analysis



As long as $F_s \geq 2B$ then we can clearly "see"…
a view of $X(f)$ in $\tilde{X}(f)$

But we "did" this using a FT of a signal inside the DAC…
Is there some other way to do this by <u>using the *samples*</u>?

Motivation for D-T Fourier Transform (DTFT)



- We know that if sampling has been done "perfectly" that:
  - $\tilde{X}(f)$ shows the original signal's FT $X(f)$ with no aliasing
  - But… that is only something that helps us "conceptually" but not really "numerically"…
- So that raises this question:
  - Since $\tilde{x}(t)$ is completely determined by $x[n]$… can we use those samples to actually ***compute*** $\tilde{X}(f)$????

Recall Fourier Transform of $\tilde{x}(t)$

$$\tilde{x}(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

$$= x(t)\delta_T(t)$$

FS of $\delta_T(t)$

$$\tilde{x}(t) = \frac{1}{T} \sum_{k=-\infty}^{\infty} x(t) e^{jk2\pi F_s t}$$

FT & Mod. Prop

$$\tilde{X}(f) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X(f + kF_s)$$

Tells what $\tilde{X}(\omega)$ looks like!

Take An Alternate Path to the DTFT!

$$\tilde{x}(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT) = x(t)\delta_T(t)$$

FS of $\delta_T(t)$

$$\tilde{x}(t) = \frac{1}{T} \sum_{k=-\infty}^{\infty} x(t) e^{jk2\pi F_s t}$$

FT & Mod. Prop

$$\tilde{X}(\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X(\omega + k2\pi F_s)$$

Tells what $\tilde{X}(\omega)$ looks like!

$$\tilde{x}(t) = \sum_{n=-\infty}^{\infty} x[n]\delta(t - nT)$$

$$\tilde{X}(\omega) = \mathfrak{F}\left\{ \sum_{n=-\infty}^{\infty} x[n]\delta(t - nT) \right\}$$

$$= \sum_{n=-\infty}^{\infty} x[n] \mathfrak{F}\{\delta(t - nT)\}$$

$$\tilde{X}(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-jn\omega T}$$

Uses Samples!!

Tells how to compute $\tilde{X}(\omega)$!

Re-Define to Get The DTFT!

$$\tilde{X}(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-jn\omega T}$$

Let $\Omega = \omega T$ where $T = 1/F_s$

DTFT:

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-jn\Omega}$$

Ω is called "D-T Frequency"

$\Omega = \omega T$: (rad/sec) × (sec/sample) = rad/sample

$\tilde{X}(\omega)$ and $X(\Omega)$ are really the same thing…

just "plotted" w.r.t. a different unit

***ω*: rad/sec**

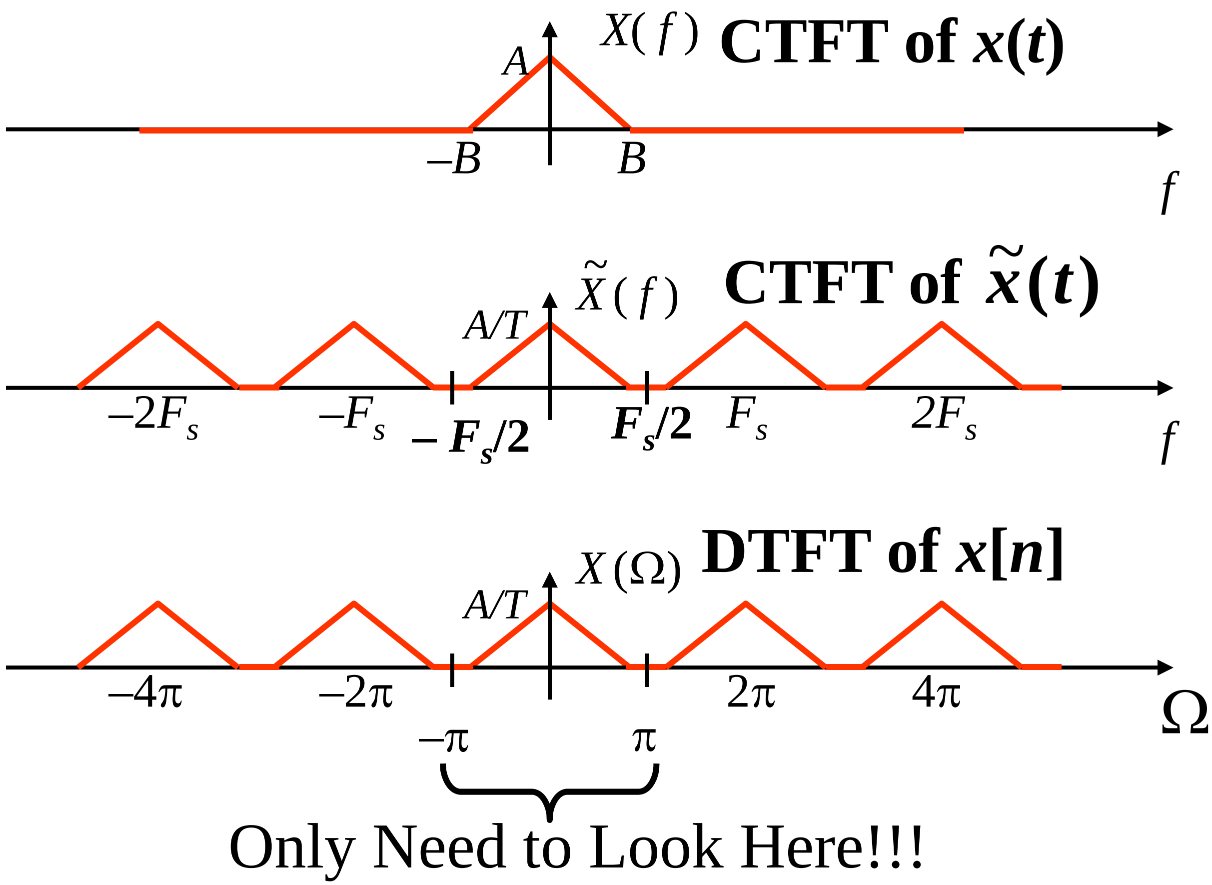
Ω: rad/sample

DTFT $X(\Omega)$ shows…

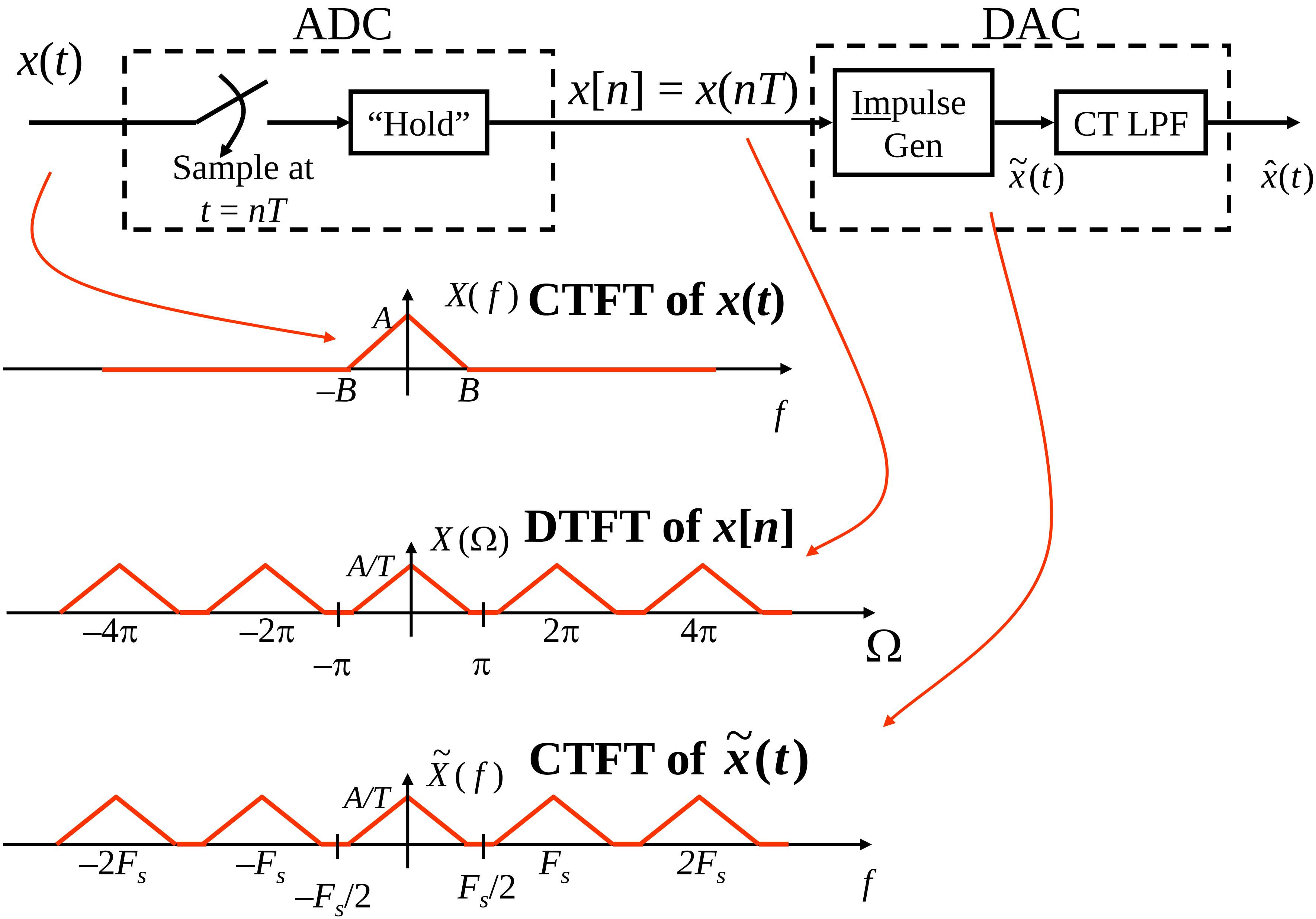
$\tilde{X}(f)$ **which shows…**

If sampling was done right!!!

"CTFT" $X(f)$



Physical Relationship of DTFT



Motivating D-T *System Analysis* using DTFT