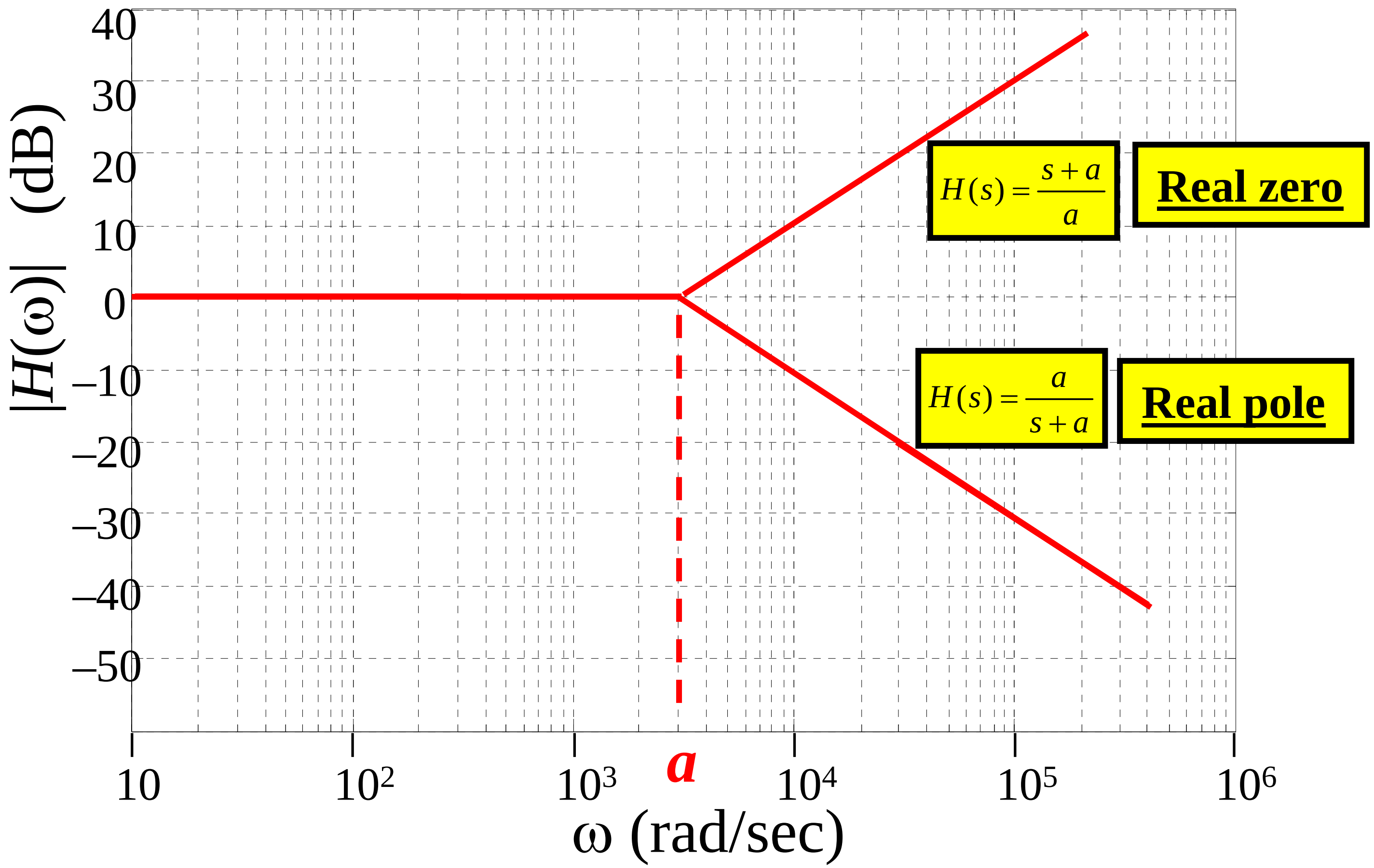
BINGHAMTON UNIVERSITY
State University of New York

EECE 301
Signals & Systems
Prof. Mark Fowler

Discussion #11

- Bode Plot Method and Example

We have seen two cases: Real Pole & Real Zero



This allows us to handle all <u>real</u> poles/zeros in the <u>left-hand plane</u>.

So we still need a way to handle two other cases.

-zero/pole at s = 0

-zero/pole complex conjugate pairs – 2[nd] order term

Zero/Pole at $s = 0$

Zero at $s = 0$

$$H(s) = s$$

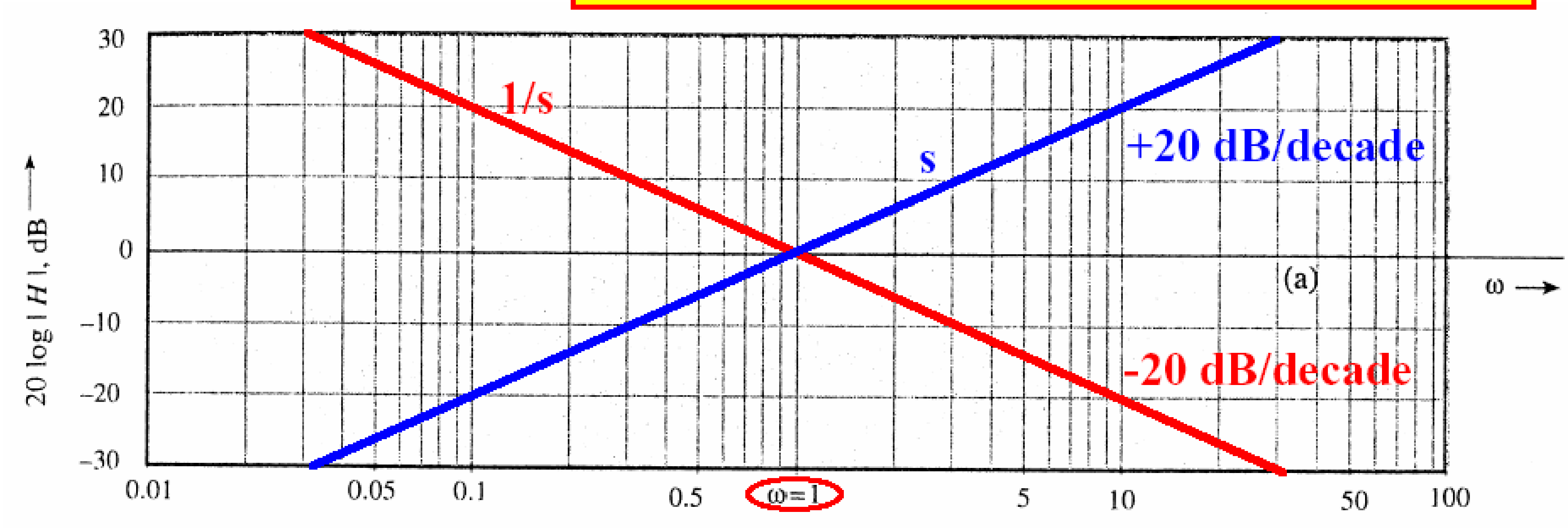
Pole at $s = 0$

$$H(s) = \frac{1}{s}$$

Replace $s \rightarrow j\omega$ and take magnitude:

$$\underbrace{\pm 20 \log_{10}(\omega) \quad vs. \quad \log_{10}(\omega)}$$

Line of slope ±20 that goes through 0dB at $\omega = 1$



Complex Conjugate Pair ($0 \le \zeta \le 1$)

General Form: Complex pair of poles Complex pair of zeros

$$\frac{\omega_n}{s^2 + 2\zeta\omega_n s + \omega_n{}^2} \qquad \frac{s^2 + 2\zeta\omega_n s + \omega_n{}^2}{\omega_n}$$

ω_n breakpoint

for $0 \le \zeta \le 1$

on $j\omega$ axis (at $\zeta = 0$)

$\zeta = 1$ gives repeated real roots.

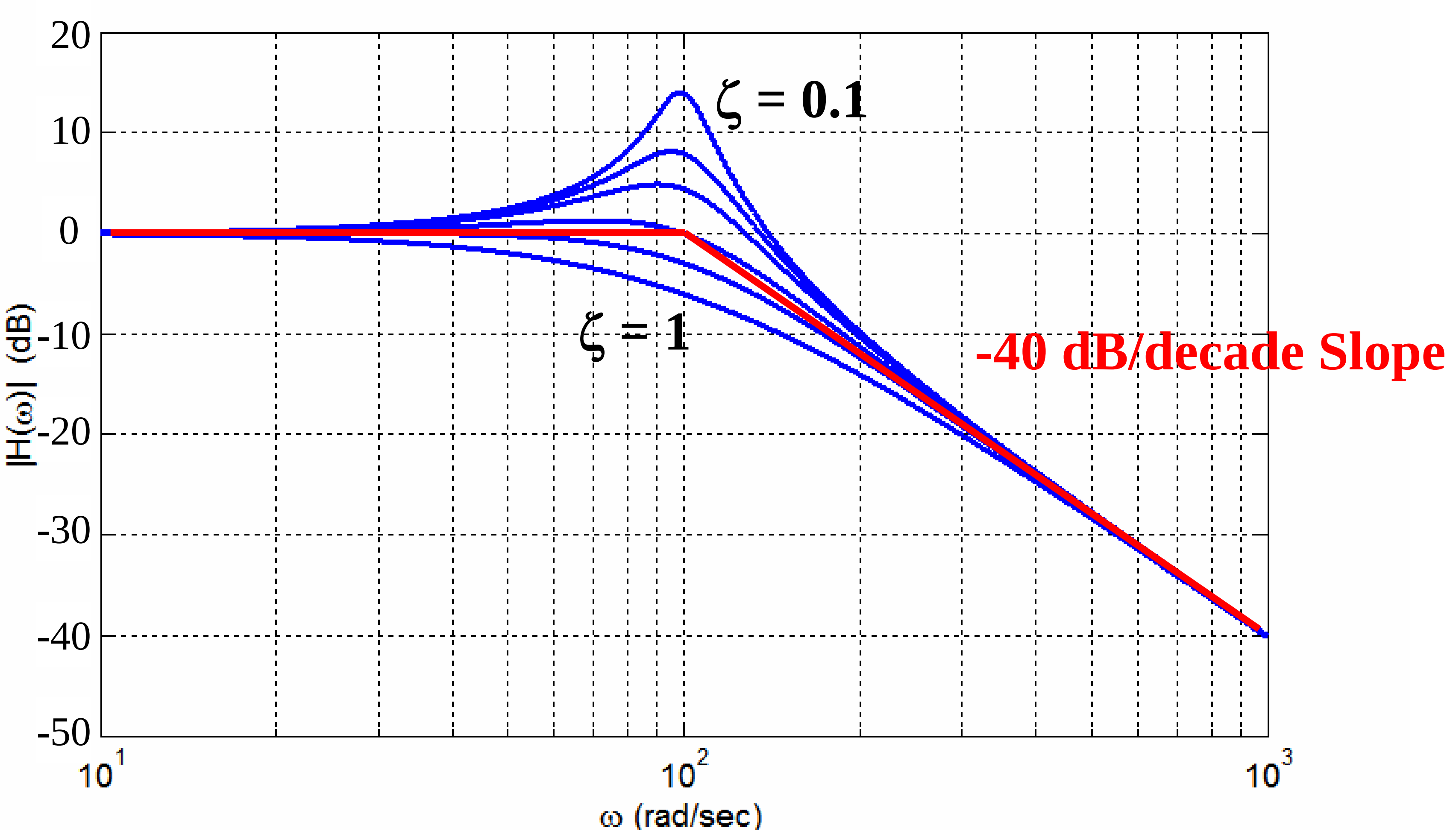
$$\Rightarrow 20\log_{10}\left[\left|\left(\frac{j\omega}{\omega_n}\right)^2 + \frac{2\zeta}{\omega_n} j\omega + 1\right|\right] \quad vs. \quad \log_{10}(\omega)$$

$\approx 0dB$ (constant) *for* $\omega << \omega_n$

$\approx \pm 40dB$ per decade *for* $\omega >> \omega_n$

For a 2nd – order pole:

(shown for ω_n = 100)

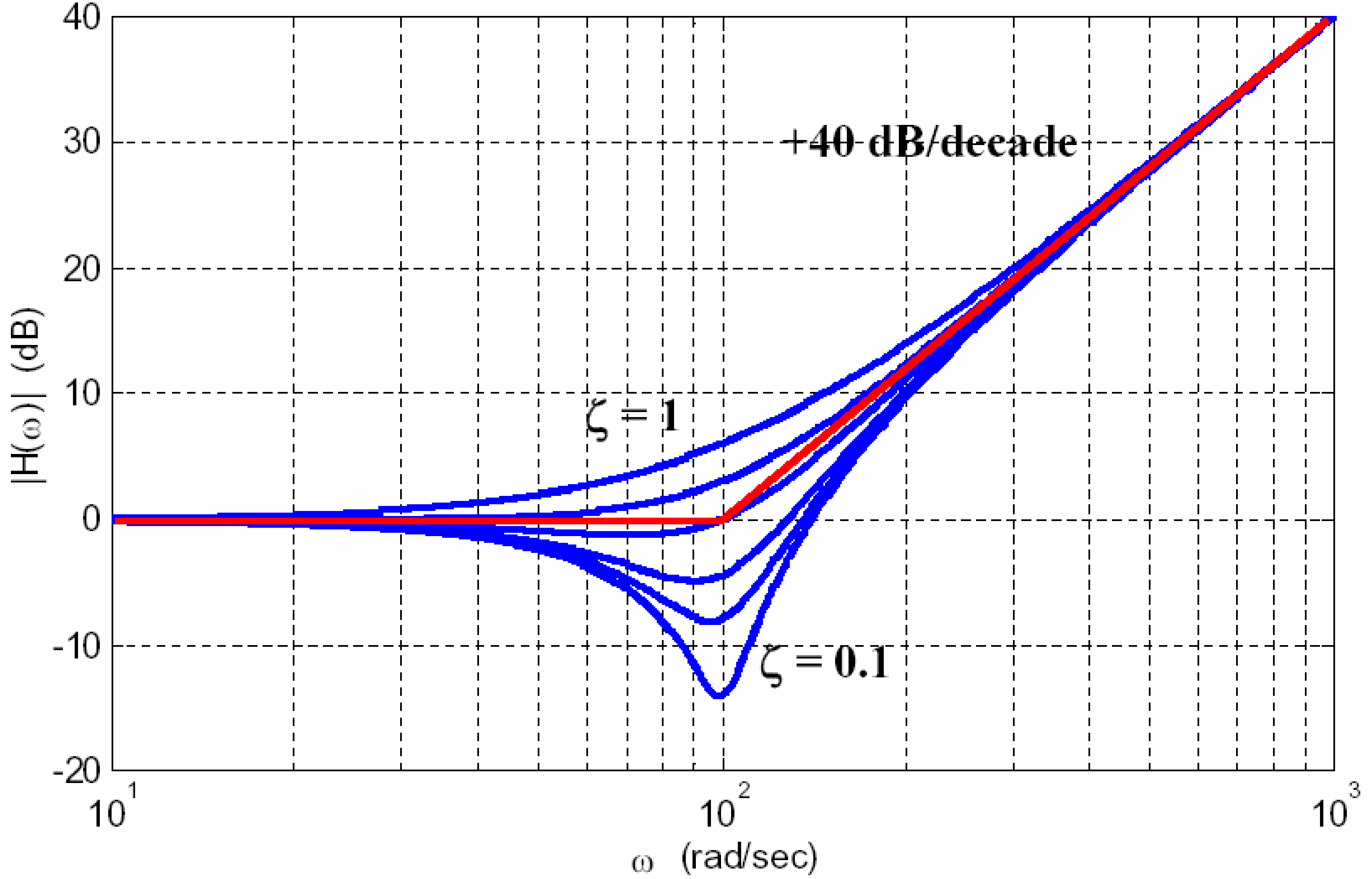


Note that as ζ gets smaller the pole gets closer to the *jω* axis…

which causes a larger peak.

For a 2nd – order zero:

(shown for $\omega_n = 100$)



Note that as ζ gets smaller the zero gets closer to the $j\omega$ axis… which causes a deeper null.

General Steps to "Sequentially" Build Bode Plots

1. Factor $H(s)$… leave complex-root terms as quadratics
2. Convert to $j\omega$ form
3. Pull out "constants" into a "gain" term
4. Combine constant term with "$j\omega$" terms (if any)
5. Identify "break points" and put in ascending order
6. Plot constant term with "$j\omega$" terms at ω values below the lowest "break point"
7. At "break point", change slope by ±20dB/decade or ±40dB/decade for 1st order or 2nd order terms, repectively.
   - Repeat this step through ordered list of "breakpoints".
8. Make "resonant corrections" for "under damped" 2nd order terms (i.e. when $\zeta < 0.5$).

Example

$$H(s) = \frac{0.1s(s+50)(s+200)}{(s+2)(s^2+2s+100)}$$

1. Already factored

$$\omega_n = 10 \Rightarrow 2\zeta\omega_n = 2$$

$$\Rightarrow \zeta = 0.1 \qquad \Rightarrow complex \quad pair$$

2. Convert to *jω*:

$$H(\omega) = \frac{0.1 j\omega(j\omega+50)(j\omega+200)}{(j\omega+2)((j\omega)^2+2j\omega+100)}$$

3. Pull Out "Constants":

$$H(\omega) = \frac{0.1 j\omega(j\omega+50)(j\omega+200)}{(j\omega+2)((j\omega)^2+2j\omega+100)}$$

$$H(\omega) = \underbrace{\frac{0.1\times 50\times 200}{2\times 100}}_{=5}\left[\frac{j\omega(1+j\omega/50)(1+j\omega/200)}{(1+j\omega/2)\left(1+2j\omega/100+(j\omega/10)^2\right)}\right]$$

Gain Term

4. Combine gain term with $j\omega$ term :

$$H(\omega)=\left[\frac{(5j\omega)(1+j\omega/50)(1+j\omega/200)}{(1+j\omega/2)\left(1+2j\omega/100+(j\omega/10)^2\right)}\right]$$

5. Identify Breakpoints and List in Ascending Order:

$$H(\omega)=\left[\frac{(5j\omega)(1+j\omega/\boxed{50})(1+j\omega/\boxed{200})}{(1+j\omega/\boxed{2})\left(1+2j\omega/100+(j\omega/\boxed{10})^2\right)}\right]$$

List breakpoints in ascending order:

Break Points	*Change* in slope
2	-20dB/decade – 1st order term in denominator
10	-40dB/decade – 2nd order term in denominator
50	+20dB/decade – 1st order term in numerator
200	+20dB/decade – 1st order term in numerator

6. Plot constant term with "$j\omega$" terms at ω values below the lowest "break point" :

-Pick ω value that is (at least) 1 decade below the lowest BP: $\omega = 0.1$

-Evaluate $|5j\omega|$ there in dB:

$$20\log_{10}(5\times 0.1) = 20\log_{10}(0.5) = -6dB$$

-Plot a point at -6 dB at $\omega = 0.1$

-Draw a line of slope 20dB/decade from this point up to the first BP

7. At "break point", change slope by ±20dB/decade or ±40dB/decade for 1st order or 2nd order terms, repectively. :

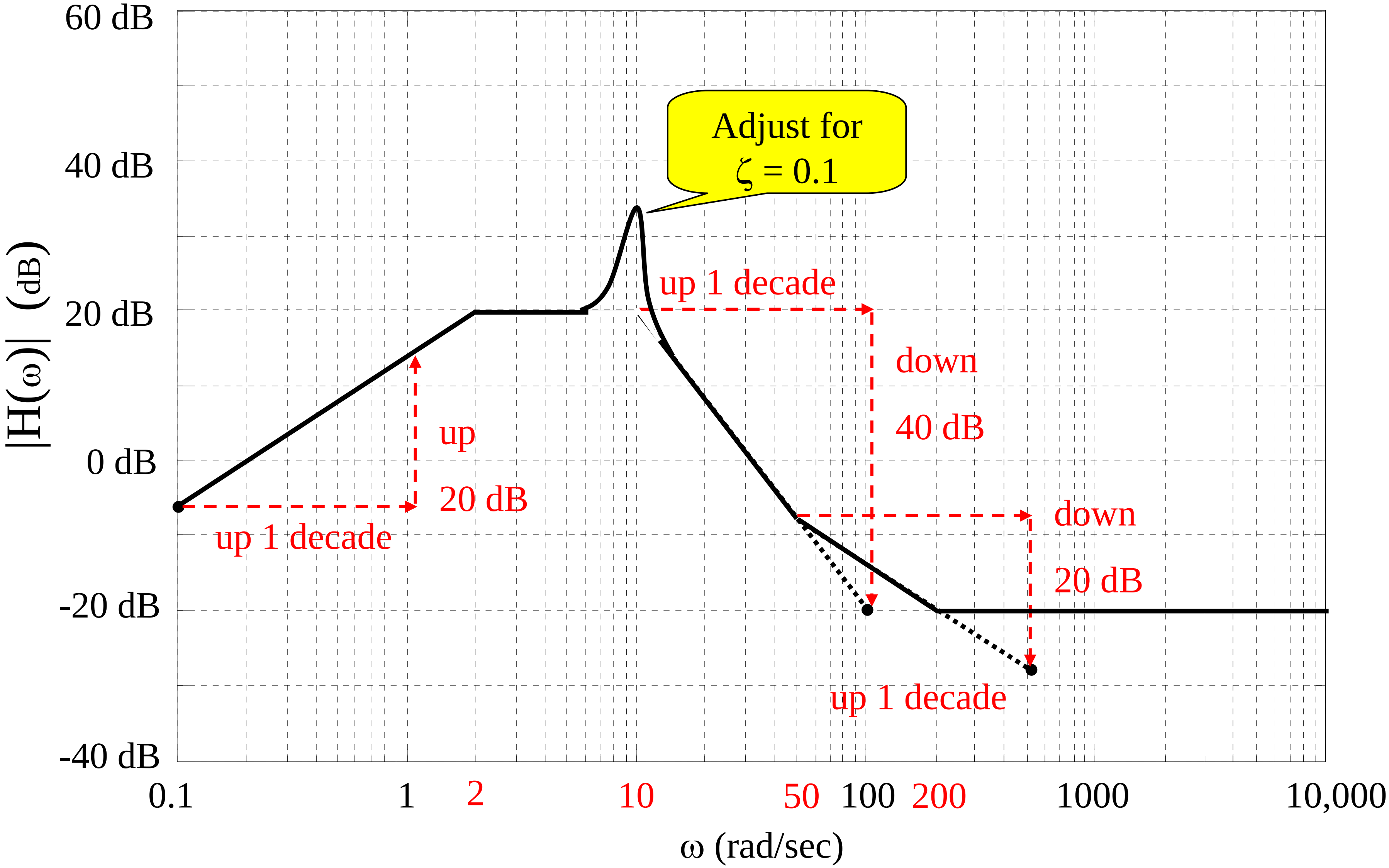
Break Points	*Change* in slope
2	-20dB/decade – 1st order term in denominator
10	-40dB/decade – 2nd order term in denominator
50	+20dB/decade – 1st order term in numerator
200	+20dB/decade – 1st order term in numerator

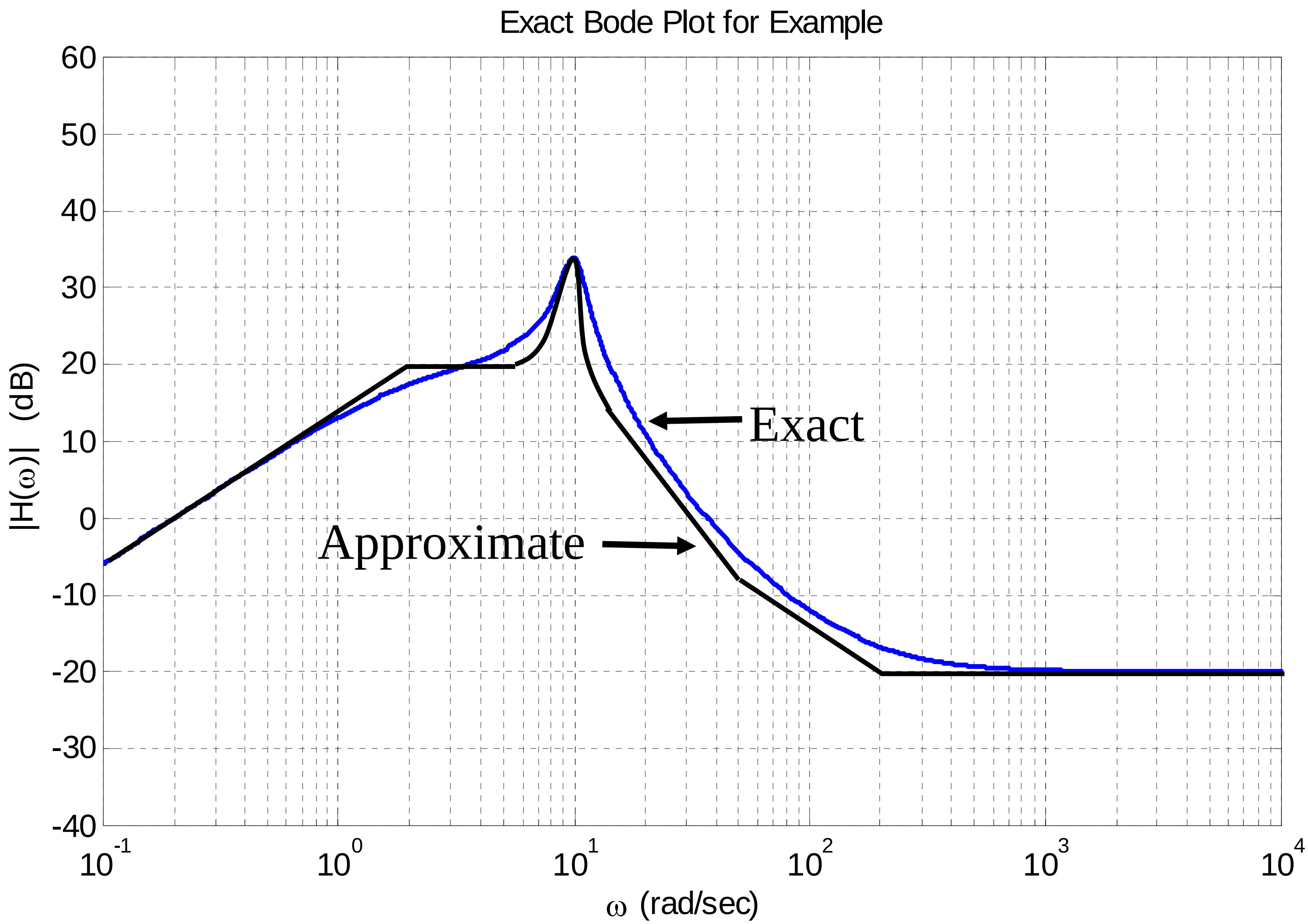
8. Make "resonant corrections" for "under damped" 2nd order terms (i.e. when $\zeta < 0.5$). :

Finally: Make adjustment for the ζ value from the plot of the 2nd order term: $\zeta = 0.1$ gives peak ≈ 14dB up

ζ value	Adjustment
0.1	14 dB
0.2	8 dB
0.3	5 dB
0.4	3 dB
0.5	1 dB

Approximate Bode Plot for Example in Notes




Exact Bode Plot for Example
60
50
40
30
20
10
0
-10
-20
-30
-40
|H(ω)| (dB)
10^-1
10^0
10^1
10^2
10^3
10^4
ω (rad/sec)
Exact
Approximate