BINGHAMTON UNIVERSITY
State University of New York

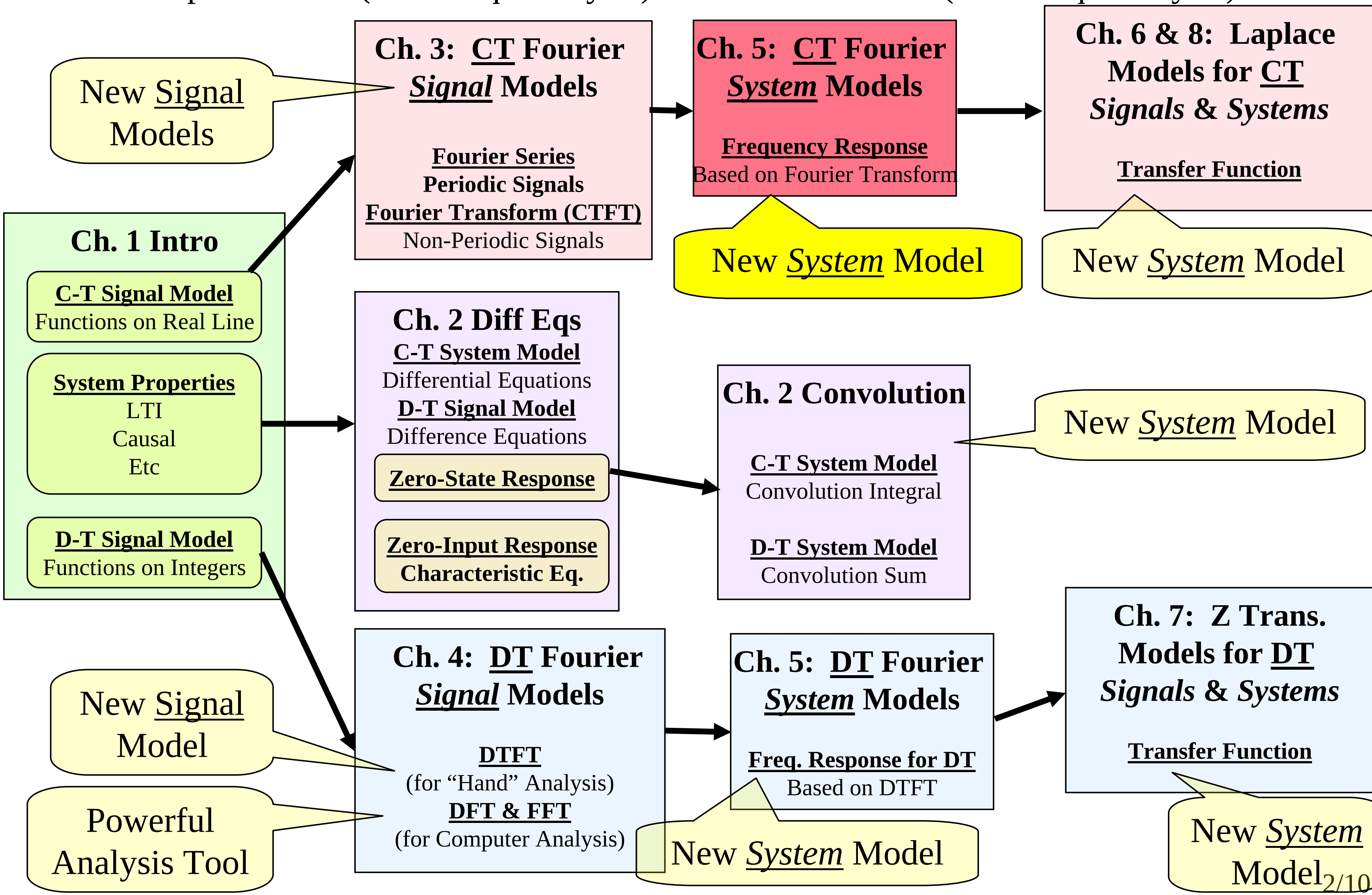
EECE 301
Signals & Systems
Prof. Mark Fowler

Note Set #17

- C-T Systems: Frequency-Domain Analysis of Systems
- Reading Assignment: Section 5.1 of Kamen and Heck
- We’re jumping over Ch. 4 for now… we’ll come back later

Course Flow Diagram

The arrows here show conceptual flow between ideas. Note the parallel structure between the pink blocks (C-T Freq. Analysis) and the blue blocks (D-T Freq. Analysis).



Ch. 5 Frequency-Domain Analysis of Systems

Our main interest in this chapter is:

How do we use the FT to analyze LTI systems?

We'll focus on the zero-state response here…

(The zero-input response can be found using the characteristic equation method or the more complete methods we'll study later)

We'll look first at CT systems using three steps:

5.1: Find out how sinusoids go through a C-T LTI

5.2: Because a periodic signal is a sum of sinusoids we use linearity to extend section 5.1 results to periodic signals.

5.2: Non-periodic signals also can be viewed as a sum (really an integral) of sinusoids so we can extend the result again!

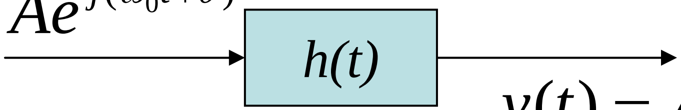
Later we'll essentially do the same things for D-T systems.

In between we'll look at "Ideal C-T Filters" and "Sampling" to convert C-T signals into D-T signals

5.1 Response to a sinusoidal input:

In the notes for Section 3.1 (when we motivated WHY we were studying FS) we saw that it is easy to state how a complex sinusoid goes through a C-T LTI system :

$$x(t) = Ae^{j(\omega_0 t+\theta)}$$



$$y(t) = Ae^{j(\omega_0 t+\theta)} \underbrace{\int_{-\infty}^{\infty} h(\tau)e^{-j\omega_0 \tau}d\tau}_{= H(\omega_0)}$$

We now know that this is the FT of the system's impulse response, evaluated at $\omega = \omega_0$

$$y(t) = AH(\omega_0)e^{j(\omega_0 t+\theta)}$$

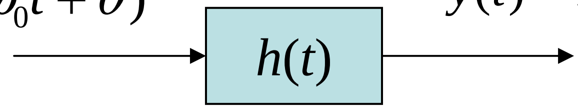
$$y(t) = |H(\omega_0)|Ae^{j(\omega_0 t+\theta+\angle H(\omega_0))}$$

Same frequency sinusoid comes out… the system just changes the input sinusoid's amplitude and phase

An LTI acts to change a complex sinusoid's amplitude and phase

We also saw how a *real* sinusoid goes through a C-T LTI System

$x(t) = A\cos(\omega_0 t + \theta)$



$y(t) = A|H(\omega_o)|\cos(\omega_o t + \theta + \angle H(\omega_o))$

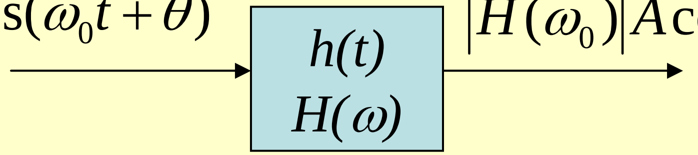
The only thing an LTI system does to a real sinusoid is change its amplitude and its phase!!!!

Of course, you already knew that from circuits!!

So… The big result is:

$h(t)$ = impulse response $\xleftrightarrow{\text{FT}}$ $H(\omega)$ = frequency response

$A\cos(\omega_0 t + \theta)$

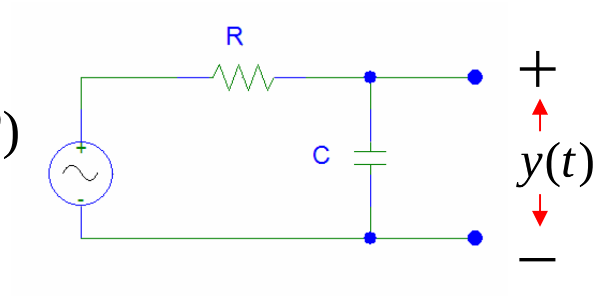


$|H(\omega_0)|A\cos(\omega_0 t + \theta + \angle H(\omega_0))$

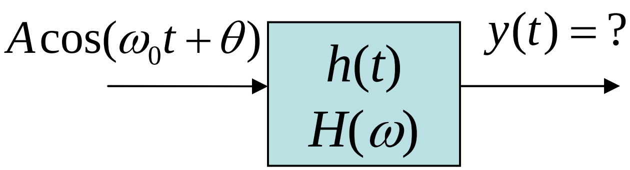
***H*(*ω*) is called the "frequency response" of the system**

<u>Example:</u> Connecting these general ideas to sinusoidal analysis of circuits.

$$x(t) = A\cos(\omega t + \theta)$$



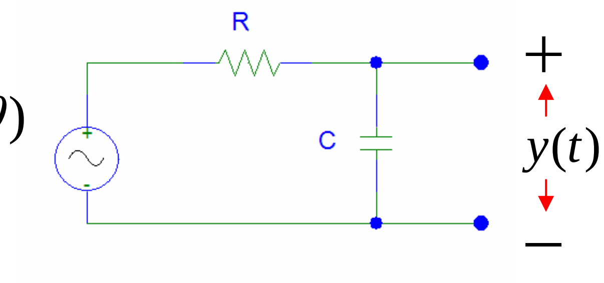
To go from the circuit view to the system view… we need $H(\omega)$



When you did sinusoidal analysis in Circuits you did this!!!

Sinusoidal Analysis of Circuit gives the System's Frequency Response $H(\omega)$

$$x(t) = A\cos(\omega t + \theta)$$



1. Convert capacitor into impedance: $Z_c(\omega) = \dfrac{1}{j\omega C}$

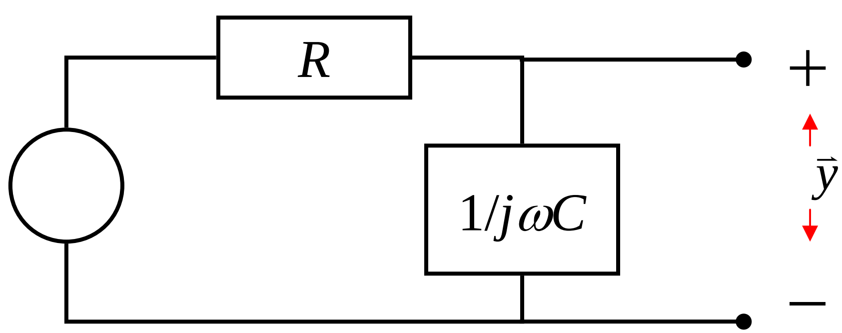
Small impedance at high ω
Large impedance at low ω

2. Write input as phasor: $Ae^{j\theta} = \vec{x}$

Phasor captures amplitude and phase of cosine… the only things the system can change!!

3. Now analyze the circuit as if it were a DC circuit with a complex voltage in (the phasor) and complex resistors (the impedances):

$$Ae^{j\theta} = \vec{x}$$



Now find the output phasor as a function of the input phasor… Here this is easiest using voltage divider!

Voltage Divider: $$\vec{y} = \frac{Z_c(\omega)}{R + Z_c(\omega)}\vec{x} = \underbrace{\left[\frac{1}{1 + j\omega RC}\right]}_{=H(\omega)}\vec{x}$$

Output Phasor: $$\vec{y} = H(\omega)\vec{x} = |H(\omega)|e^{j\angle H(\omega)}\vec{x}$$

$$= |H(\omega)|e^{j\angle H(\omega)}Ae^{j\theta}$$

$$= \left(|H(\omega)|A\right)e^{j(\theta + \angle H(\omega))}$$

4. Convert the "phasor solution" into the "sinusoidal solution":

Remember that a phasor is a complex number that holds:

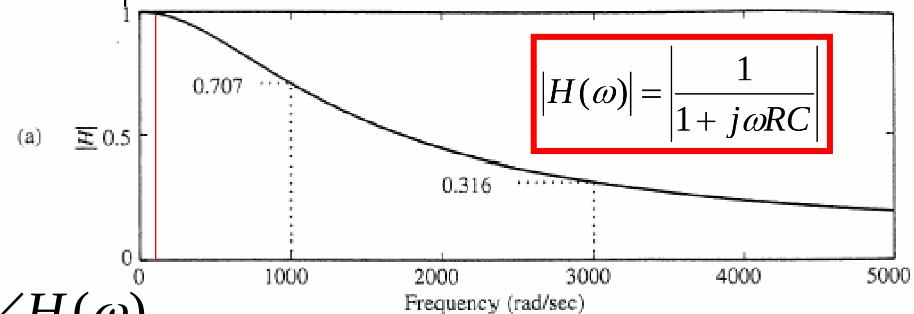
- sinusoid's amplitude in its magnitude
- sinusoid's phase in its angles

$$\vec{y} = \overbrace{\left(|H(\omega)|A\right)}e^{j\overbrace{(\theta + \angle H(\omega))}} \quad \Rightarrow \quad y(t) = \overbrace{|H(\omega)|A}\cos(\omega t + \overbrace{\theta + \angle H(\omega)})$$

To see how different frequencies are affected by the RC circuit we plot

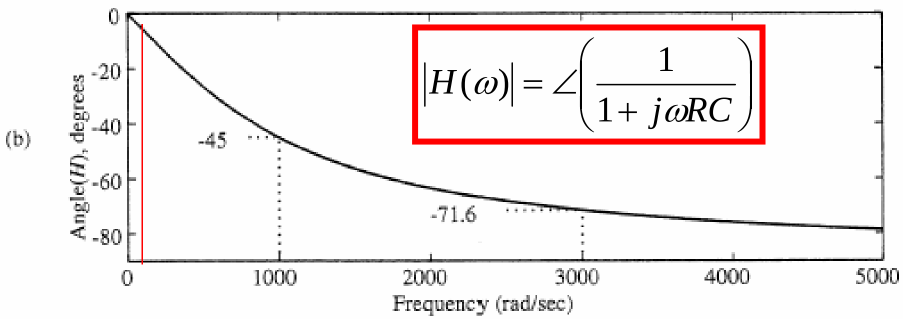
$$|H(\omega)| \;\&\; \angle H(\omega)$$

$|H(\omega)|$



$$|H(\omega)| = \left|\frac{1}{1+j\omega RC}\right|$$

$\angle H(\omega)$



$$|H(\omega)| = \angle\left(\frac{1}{1+j\omega RC}\right)$$

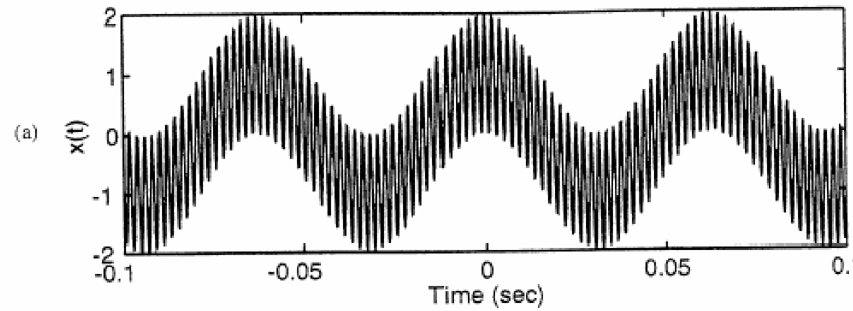
Figure 5.2 (a) Magnitude and (b) phase functions of the *RC* circuit in Example 5.2 for the case $1/RC = 1000$.

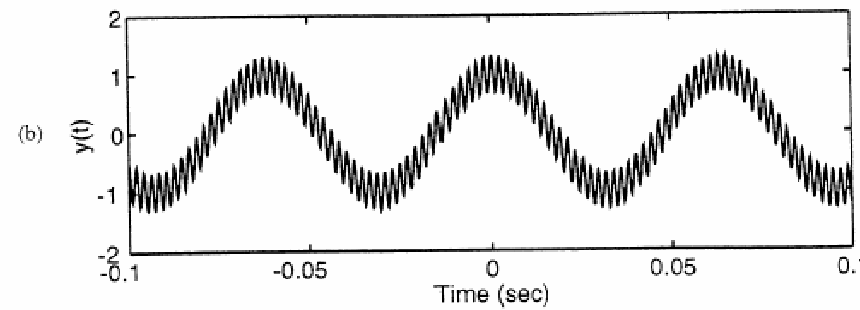
$$H(100) = 0.995e^{-j0.097}$$

$$H(3000) = 0.316e^{-j1.249}$$

Input has equal amounts at the 2 frequencies…

$$x(t) = \cos(100t) + \cos(3000t)$$





$$y(t) = 0.995\cos(100t - 0.097) + 0.316\cos(3000t - 1.249)$$

Output has almost all of the low frequency component but much reduced high frequency component!

So what have we seen:

- We can find the frequency response function $H(\omega)$ by doing a simple sinusoidal analysis of the circuit
- The frequency response function tells how a circuit changes the input sinusoid's amplitude and phase
- The amount of change in each of these is different for different input frequencies… and a plot of $H(\omega)$ magnitude and phase shows this dependence
- RLC circuits can be used to allow certain frequency components to pass mostly unchanged while others are drastically reduced in amplitude
  - We can "filter out" undesired frequency components